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TRANSIENT TEMPERATURE MEASUREMENT ERRORS

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ABSTRACT

The estimation of transient temperature measurement errors is often required to help understand thermal experiments and improve the accuracy of estimated thermal parameters. Thermal response models used in conjunction with experimental techniques are very effective. A hybrid of finite differences and the Unsteady Surface Element method is developed and used for modeling temperature measurements made with intrinsic thermocouples and resistance temperature detectors. In the latter case, experimental data obtained with the Loop Current Step Response method is used to estimate model parameters.

INTRODUCTION

In many thermal experiments, an understanding of transient temperature measurement errors is required in the interpretation of the results or in the data analysis to determine process parameters. The accuracy of these measurements can have a significant effect on the predicted value of the parameters. Thermal modeling of the sensor installation can give significant insight into the experiment itself by providing estimates of the errors involved and their time dependence. Such modeling is useful in the development of the framework for analyzing the experimental data.

Many of the problems in obtaining accurate transient thermal measurements stem from a lack of an adequate response model and the lack of dynamic sensor calibration. A significant amount of work has been done to overcome these problems; a few of the related papers are listed [1-16].

A combination of experimental and analytical techniques is often effective because one can be used to refine the other. In this paper an analytical technique called the Unsteady Surface Element Method [8] is utilized with an experimental technique called the Loop Current Step Response (LCSR) method [14].

The Unsteady Surface Element (USE) Method and related methods have been used to analyze both thermal sensors and installation effects [5,6,11,12,16]. When applicable, the method is simple to use, and in some instances, the results can be expressed in analytical form [8,15]. The USE method as developed in [8] imposes certain restrictions on the allowed geometries. To overcome some of these restrictions, a hybrid method which combines the USE method with a simple finite difference model is developed.

The Finite Difference/Unsteady Surface Element (FUSE) Method maintains most of the simplicity of the USE model. In certain instances, analytical solutions are still obtainable albeit more complicated. In general the increased flexibility is accompanied by increased numerical complexity although it is still simpler than most numerical methods. Generally it is simple to program; some problems have been solved on programmable calculators. It can often be used on problems which pose difficulties for numerical techniques, such as semi-infinite domains and/or very small or very long times.

In the Loop Current Step Response Method [14], a heating current is sent through the sensor element to produce a temperature transient. For resistance temperature detectors either the rise or the decay (which occurs when the current is turned off) can be used. For thermocouples, in general, only the decay can be used. The LCSR method produces a response to internal heating; this needs to be related to the response for an external stimulus. While this has been done analytically for the specific class of sensors in which the heat transfer is one-dimensional [14], it has not been accomplished for sensors with significant multidimensional conduction paths.

In this paper, a brief development of the FUSE method will be given and then used in two applications. For a flash diffusivity experiment, the effect of welding induced junction distortion for intrinsic thermocouples will be considered and compared with experimental results. For a platinum RTD which is adhesively mounted on a plate, data from LCSR tests

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will be used to help define the appropriate FUSE model. The model will be used to predict response to an external forcing function.

FUSE MODEL DESCRIPTION

The USE method was developed to calculate the interface temperature or heat flux beteen two arbitrary connected bodies. In the USE method the coupling of two bodies, at one or more nodes, is considered exact for either temperature or heat flux at each node and is satisfied only on the average at each node for the second condition, heat flux or temperature. An equation involving Duhamel's integral with a kernel function for either a step change in temperature or heat flux is written for each body. The interface boundary conditions are used to couple the two equations.

For simplicity, the case to be considered involves two arbitrary bodies with single surface contacts and a one-dimensional finite difference mesh. The equations can be simplified to the case of a single arbitrary body or where one or both of the bodies are replaced by a well-stirred fluid.

The configuration generally considered is shown schematically in Fig. 1. The nodal structure utilizes T-lumping. The resistance elements incorporate both the conduction resistance in the nodes and contact resistance at the interfaces between nodes or between a node and a body. The capacitance depends upon the node material. If there is a contact resistance or convective type boundary condition at the contact with a body, a π-lumping can be used adjacent to the body.

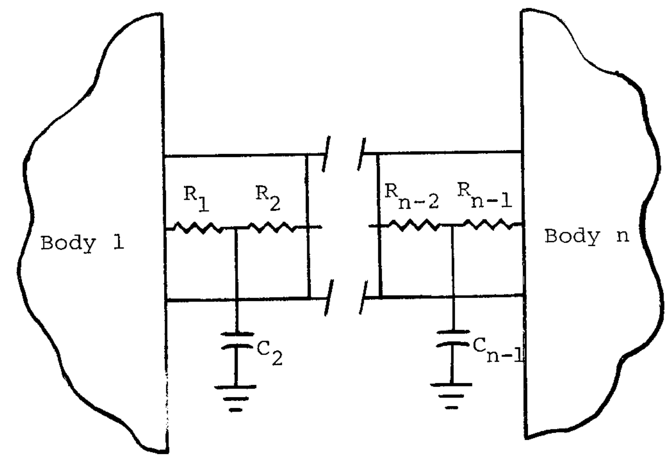

Fig. 1 Schematic of a FUSE model

Both the bodies and the nodes may be set at individual initial temperatures. The bodies may have time-dependent volumetric heating, while the nodes may have time-dependent heat generation terms. If one or both of the bodies is finite, there may be a boundary condition imposed on the other surfaces.

The equations for the temperatures at the contact surfaces of the bodies and at the nodes are developed with the heat-flux based USE method [8]. They are given for the case where there is no imposed boundary condition on the bodies.

For the contacting area of Body 1, Duhamel's integral gives

$$T_1(t) = T_{1i} + \frac{\partial}{\partial t}\int_0^t g_1'''(\lambda)(t-\lambda)/\rho_1 c_1 \, d\lambda + \frac{\partial}{\partial t}\int_0^t q_1(\lambda)\phi\,(t-\lambda)\,d\lambda \qquad (1)$$

where $q_1(t)$ is the heat flux into the contact on body 1, $g_1'''(t)$ is a volumetric heat generation, and ϕ_1 is the average contact temperature resulting from a unit step change at t=0 in heat flux over the contact area.

Using the difference formulation of Fourier's equation, the heat flux q_1 can be written as

$$q_1 = -k\Delta T/\Delta L = (T_2-T_1)/R_1A_1 \qquad (2)$$

where R_1 is the thermal resistance between the body surface and node 2.

Using central differences for the spatial terms, the differential-difference equation for the temperature of node 2 is given by

$$C_2dT_2(t)/dt = -[T_2(t)-T_1(t)]/R_1+[T_3(t)-T_2(t)]/R_2 + g_2(t) \qquad (3)$$

where $g_2(t)$ is the heat generation term in node 2.

Similar equations can be written for other interior nodes,

$$C_3dT_3(t)/dt = -[T_3(t)-T_2(t)]/R_2+[T_4(t)-T_3(t)]/R_3 + g_3(t) \qquad (4)$$

$$C_{n-1}dT_{n-1}(t)/dt = -[T_{n-1}(t)-T_{n-2}(t)]/R_{n-2}+[T_n(t) - T_{n-1}(t)]/R_{n-1}+g_{n-1}(t) \qquad (5)$$

Analogous to Eq. (1) for body 1, Duhamel's integral for body n gives

$$T_n(t) = T_{ni} + \frac{\partial}{\partial t}\int_0^t g_n'''(\lambda)((t-\lambda)/\rho_n c_n)\,d\lambda - \frac{\partial}{\partial t}\int_0^t q_n(\lambda)\phi_n(t-\lambda)\,d\lambda \qquad (6)$$

The thermal properties are all considered temperature-independent as required by the linearity restriction of the convolution equations.

If the Laplace transform of the above set of equations is taken and then put in matrix notation, the result is

$$[A][\bar{T}] = [\bar{T}_{ji}] + [\bar{g}] \qquad (7)$$

where

$$
[A] = \begin{bmatrix}
1+s\bar{\phi}_1/R_1A_1 & -s\bar{\phi}_1/R_1A_1 & & & & \\
-1/R_1 & C_2s+(1/R_1+1/R_2) & -1/R_2 & & & \\
& \cdot & \cdot & \cdot & & \\
& & \cdot & \cdot & \cdot & \\
& & & -1/R_{n-2} & C_{n-1}s+(1/R_{n-2}+1/R_{n-1}) & -1/R_{n-1} \\
& & & & -s\bar{\phi}_n/R_{n-1}A_n & 1+s\bar{\phi}_n/R_{n-1}A_n
\end{bmatrix} \tag{8}
$$

$$
[\bar{T}] = \begin{bmatrix} \bar{T}_1 \\ \bar{T}_2 \\ \vdots \\ \bar{T}_{n-1} \\ \bar{T}_n \end{bmatrix} \qquad
[\bar{T}_{ji}] = \begin{bmatrix} T_{1i}/s \\ C_2T_{2i} \\ \vdots \\ C_{n-1}T_{n-1,i} \\ T_{ni}/s \end{bmatrix} \qquad
[\bar{g}_j] = \begin{bmatrix} \overline{g_1'''}/\rho_1c_1s \\ \bar{g}_2 \\ \vdots \\ \bar{g}_{n-1} \\ \overline{g_n'''}/\rho_nc_ns \end{bmatrix} \tag{9}
$$

Setting up the problem is relatively straightforward. The meshed portion is constructed utilizing as few nodes as is consistent with the time resolution required for the problem at hand. Specifying the bodies may be more complex. The ϕ functions used in Equations (1) and (6) or the first and last rows of matrix form, Eqn. (7), are defined as the average temperature over the contact area which results from a uniform step change in heat flux over the area. If a body can be considered as a flowing liquid or an infinite capacity well-stirred fluid, then the temperature is unchanged by the heat flux and thus $\phi(t) = 0$. If the body is a finite capacity well-stirred fluid, then $\phi(t)$ has the form of time, t, divided by thermal capacity ρcV. If the body is a solid, then $\phi(t)$ is determined from a solution of the diffusion equation for the body. For example, Ref. 8 gives results for uniform disk shaped sources on a semi-infinite body; the exact solution has the form

$$\bar{\phi}(t) = f[I_1(s)-L_1(s)] \tag{10}$$

with early and late time approximations of

$$\bar{\phi} = \frac{a}{k}[(\alpha/a^2s^3)^{1/2} - 2\alpha/\pi a^2s^2]$$

$$\bar{\phi} = \frac{a}{k}[8/3\pi s - a/2(\alpha s)^{1/2}] \tag{11}$$

There are two approaches that can be used to solve for the temperature; a symbolic form to define a transfer function type of solution or a numerical inversion at multiple values of s. The symbolic form may allow for solution of the poles and zeroes of the transfer function form which is

$$\bar{T}_j = G(s)/H(s) \tag{12}$$

where H(s) is the determinant in symbolic form of the matrix [A]. If the poles and zeroes can be determined directly, they can be utilized with parameter estimation programs for fitting experimental data such as those used in Loop Current Step Response testing of thermal sensors and in step or frequency analyses.

The numerical evaluation approach is useful for generating predictions for arbitrary conditions and for comparison with experimental data.

Note that the system of equations is tridiagonal and that all of the diagonal terms and two of the off-diagonal terms depend on s, the Laplace transform parameter.

APPLICATIONS

A number of papers have considered the problem of bead size affecting the transient response of a thermocouple [3,9,10,11, 15]. Maglic and Marsicanin [9] were concerned with this problem in connection with a flash thermal diffusivity measurement. They postulated that increased weld bead size in spark welded intrinsic thermocouples would slow the response and result in errors in the estimated thermal diffusivity values.

The experiment involved a 10 mm diameter by 5 mm long cylinder of pure iron. Two chromel-alumel thermocouples were spark welded to one flat surface. It was indicated that the size of the weld bead depended upon the thermal diffusivity of the substrate and the electrical contact resistance prior to capacitor discharge. A figure indicated that the bead diameter could be up to three times the wire diameter.

The thermocouple geometry used here assumes a hemispherical bead whose diameter could be varied. The bead is modeled by three equal thickness nodes and perfect contact is assumed between the bead and the substrate. The model is shown schematically in Fig. 2.

In a flash diffusivity experiment the backface temperature is assumed to vary as [17]

$$T(t) = 1+2\sum_{n=1}^{\infty}(-1)^n\exp(-n^2\pi^2\alpha t/L^2) \tag{13}$$

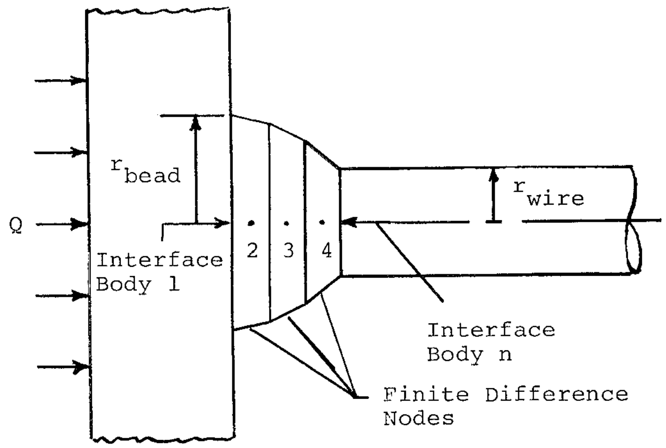

Fig. 2 Schematic of the FUSE model for the flash diffusivity experiment (certain scales distorted for clarity)

The diffusivity is generally calculated from an equation of the form

$$\alpha = k_x L^2/t_x \tag{14}$$

where k_x is a constant corresponding to x percent of the total temperature rise and t_x is the elapsed time to x percent rise. In Ref. [17], the values given for 20%, 50%, and 80% are

x	k_x
20%	.08425
50%	.1388
80%	.2332

Typically, $t_{1/2}$ is used and the total time of the temperature recorded is 4-5 times $t_{1/2}$.

The relation given by eq. (13) was used as a forcing function for the thermocouple response model (i.e., an undisturbed substrate temperature). Because the calculations showed little difference (<1% of full scale) between the two junctions, the results for the slower (i.e., alumel/iron) junction are shown in Fig. 3 for bead diameters equal to 1 and $\sqrt{10}$ times the wire diameter.

The results show that while both thermocouples lag behind the undisturbed temperature, the thermocouple with the larger bead lags slightly less than the one with no enlargement. This surprising result is believed to be due to the effects of the increased contact area more than offsetting the capacity effects of the large bead. Note that the intrinsic thermocouple junction area is increasing while the heat loss path into the wire remains fixed.

In Ref. 9, the reported diffusivity of iron at 426°C was 0.0975 cm^2/s. The $t_{1/2}$ value measured with a radiation detector was 0.355s. The $t_{1/2}$ values obtained from the four thermocouple pairs were 0.322s, 0.340s, 0.408s, and 0.444s. Two of these values are faster than the $t_{1/2}$ value calculated from Eqn. 13 for $\alpha = 0.0975$. The other two values are much longer and both thermocouples pairs involve the same iron/alumel junction.

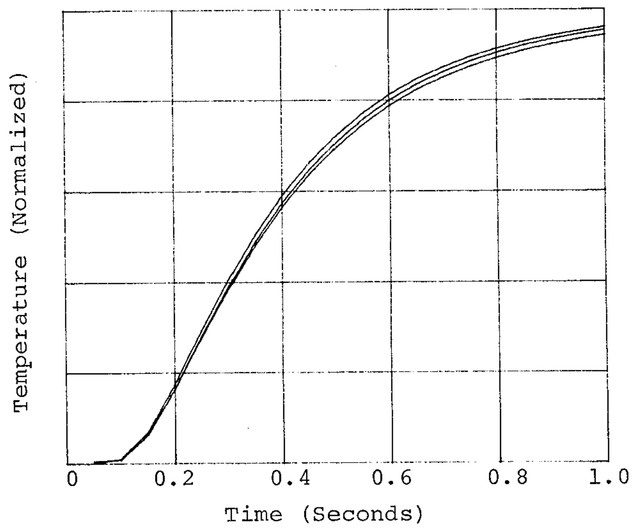

Fig. 3 Temperature histories for a flash diffuvisity experiment

Upper curve - Undisturbed backface temperature
Middle curve - Predicted temperature - enlarged bead
Lower curve - Predicted temperature - no enlargement

The calculated temperature difference between the various sized chromel/iron and alumel/iron junctions is small (~1% of the step) if perfect contact is assumed to result from the welding process. Thus, thermoelectric effects in the iron substrate will be small [12]. However, the experimental data indicates that the temperature difference between the two alumel/iron junctions is approximately one half the total temperature rise after one half second and decays to nearly zero after two seconds. These results indicate that the lag determined experimentally in Reference 9 is due to effects other than bead enlargement, possibly a very porous or cracked weld.

When a value of $\alpha = 0.1$ cm^2/s, and L = 0.5 cm was used in Eqn. 13 for the forcing function, the $t_{1/2}$ obtained from the calculated thermocouple response for no enlargement and an enlargement of $\sqrt{10}$ were .36s and .356s, respectively, as compared to the value of 0.347s from Eqn. 13. If these times are used in Eqn. 14 to calculate the diffusivity, the values of .0964 and .0975 are in error by 3.6% and 2.5% respectively. In [9], the reported error in the diffusivity from using very carefully made thermocouples with little enlargement was 3% when compared with the values calculated from an optical temperature measurement.

The FUSE model provides accurate estimates of the errors involved in using thermocouples for the flash diffusivity measurement. It provides a simple method of assessing the effects of variations in the experiment, such as the size of the weld bead.

RTD EXAMPLE

Another example of the application of the FUSE method is in the modeling of the transient response characteristics of resistance temperature detectors (RTD's). A typical problem facing an experimenter involves understanding how a sensor will respond to unsteady temperature environments after installation in a test fixture. For RTD's that are surface mounted using an epoxy type adhesive, large variations in response can be caused by apparently minor differences in how the sensor was bonded. Epoxy thickness, resin-hardener ratio and surface preparation play major roles in determining sensor response. The FUSE method, used in combination with an experimental technique known as the Loop Current Step Response method (LCSR), can be used to estimate the parameters that will control a given RTD's transient response.

The Loop Current Step Response method has been described in references [14] and [19]. For use with RTD's, it involves putting a relatively large current (20-100 mA) through the sensing element in order to induce a transient change in the sensor's resistance. The input is essentially a step change in heat generation. The functional variation of the transient is indicative of the response characteristics of the sensor in the given experimental setup. One of the chief advantages of the technique is that it is in situ; if the boundary conditions aren't changed, all the factors that will effect the sensor's response during an experiment will also effect the transient that results from the LCSR test. One of the disadvantages of the technique is that the analytical evaluation of the sensor's response, based on the LCSR transient, requires that the boundary condition be a constant temperature. While that condition is met when the sensor is in a well-stirred fluid, it no longer holds when it is mounted on a plate. The FUSE method is used to overcome that deficiency by providing a body function that properly describes the actual boundary condition of the mounted sensor.

The utility of this combined approach depends upon how adequately the one-dimensional FUSE model describes the heat transfer in the sensor installation. Because no sensors are entirely one-dimensional, the type of information obtained from the FUSE/LCSR method will vary. In some cases, quantitative values are obtained that describe a sensor's response to external perturbations. In others, qualitative information is produced that can yield an understanding of the parameters that effect a sensor; such as how its installation has altered its response rate.

The RTD used in the experiment is shown in Fig. 4; it is similar to the Rosemount 118VA sensor. The sensor was bonded to a 1/4 inch thick stainless steel plate using a silver filled epoxy. Due to the internal details of the RTD, a four node model was used in the FUSE analysis: three for the sensor and one for the epoxy bond. In addition, a semi-infinite body function was used to model the plate. Initial nodal data, such as thermal resistance and capacitance, were calculated based on available material property data.

The experiment involved recording the thermal transient produced by the step change in current used in the LCSR method. A typical data set is shown in Fig. 5. The parameter estimation process involved using initial values based on available physical information such as material property data and expected dimensions. The parameters were then optimized by adjusting each value until the model's response agreed well with the experimental data set. The results in Fig. 5 show that excellent agreement is possible between the experimental transient and the simple FUSE model. The residual plot (i.e., the model data minus the experimental data) is also shown in the

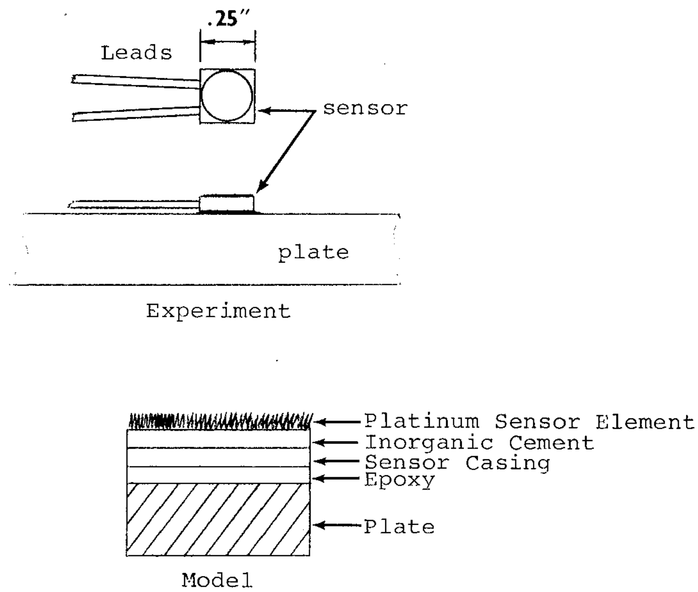

Fig. 4 Experiment setup used for LCSR test and a schematic showing the nodes used in the FUSE model

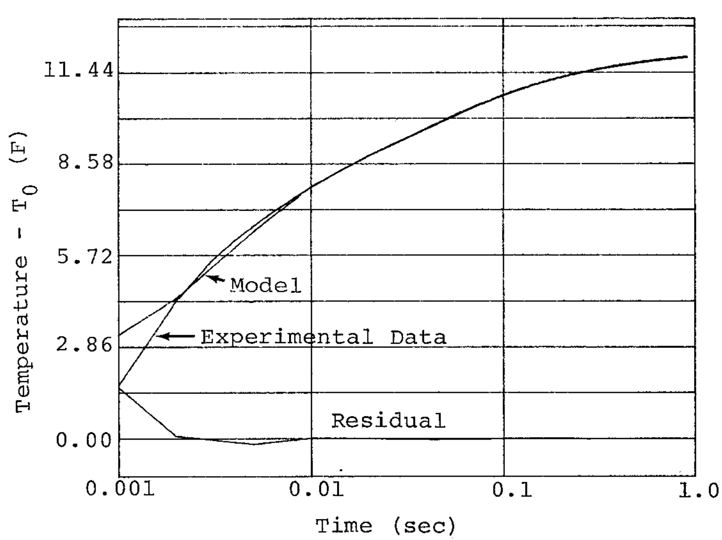

Fig. 5 Results of FUSE/LCSR technique for the response of an RTD on a plate. The upper traces are a comparison of the FUSE model to the experimental LCSR data. The lower trace is the residual or the difference between the model and the experimental data.

plot. The residual indicates that the model is only deficient in the short time region. Although the LCSR test excites the faster modes, the slower the modes are more important to accurately model the sensor response to external stimuli. This occurs because the intermediate layers (i.e., the bond, case and cement) act as filters between the external stimulus and the sensor element.

The model data produced by the parameter estimation process are useful in understanding a sensor's performance. The internal thermal resistance of the sensor appears to be almost entirely due to contact resistances between the adjacent layers that compose the sensor. The contact resistance contribution in the model was typically one to two orders of magnitude greater than the resistance to conduction in the materials in each layer. The materials used in the sensor's construction have a large effect on the transient response due to the thermal capacitance of the various layers. The application of the FUSE modeling technique to different, but supposedly identical sensors showed that large changes in the response could be accounted for by minor changes in the sensor's dimensions or material properties, for example. The performance of the sensor is also sensitive to the epoxy used in attaching the sensor to the surface. Here, the thickness of the epoxy layer and the additional contact resistances between the sensor and the surface are the important parameters.

Using the estimated model parameters, the response of the sensor to various forcing functions may be calculated. For example, the manufacturer provides plunge test data for the sensor in still water. Although the actual boundary conditions are complex, the test can approximate the step response of the sensor in water. The reported time was 0.16 seconds for a 63.2% response time. After the FUSE model was changed to remove the epoxy layer and the plate, the same response time was obtained for a convective boundary condition equivalent to a flow rate of 0.09 m/s. The response time proved to be very sensitive to the assumed flow condition. Obviously, values reported by different manufacturers would be dependent on the techniques, such as insertion speed and vibration, used in the test.

When either the one-dimensional FUSE model or the manufacturer's time constant was used to estimate the response of a mounted sensor to an external stimulus, the predicted temperature lags were significantly smaller than those measured. Additional experiments indicated that multidimensional heat conduction was the probable cause of these differences. Thus the plunge and step response data should not be interpreted as analytical approximations to experimental tests. The information is more useful when used to qualitatively predict the way in which a sensor's boundary condition can significantly alter its response characteristics.

SUMMARY

A hybrid of the Unsteady Surface Element method and a simple finite difference model has been developed. It was used to analyze the transient thermal measurements involved in two experiments. In the first case, a model was developed for intrinsic thermocouples used in a flash diffusivity experiment. It had been postulated that variations in the weld junction size seriously distorted the observed response. The analysis indicates that this assumption is incorrect and that some other effect is the cause. In the second case, a model was developed for a resistance temperature detector adhesively bonded to a plate. The model parameters which produced a best fit were obtained in a manual parameter estimation process by comparison with data from a Loop Current Step Response test. These parameters were then used to predict the response to one experiment with good results.

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