... Measurement and Control in Science and Industry, Vol. 3, Part 2

61. Heat Flux Probe for Dynamic Measurements in High Temperature Gases*

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INTRODUCTION

The heat flux probe has been developed for measuring rapid fluctuations in fluid properties over a wide range of temperatures. An internally cooled sensing element permits measurements in temperatures above the melting point of the materials used. Electrical compensating circuitry permits a low "effective" heat capacity to permit frequency responses in the kilocycle range under most operating conditions.

The instrument measures the rate of heat transfer between the environment and the internally cooled sensor. The method of measuring this heat transfer, and procedures for obtaining temperature from this information, are of primary interest. Both steady-state and transient measurements are considered along with a discussion of the errors inherent in the system.

Principle of Operation

The heat flux probe is very similar in operation to the hot wire anemometer with constant temperature compensation. The wire is replaced by a small glass tube with a platinum film on the external surface (Fig. 1). The present equipment uses a tube 0.15 millimeter outside diameter and 0.10 millimeter inside diameter with a platinum coating about 1000 A thick. Water is forced through the sensor as a coolant. The maximum heat flux the sensor can withstand depends on the rate heat is removed from the surface by the interior cooling.

The operation of the system depends on the following heat balance which can be written for the probe surface: power to probe from circuitry = heat transferred from probe surface to environment + heat transferred from probe surface to internal coolant.

In equation form:

$$I_p^2 R_p = h_0 S_0 (\theta_s - \theta_e) + S_0 U (\theta_s - \theta_w) \quad (1)$$

The compensating circuit is designed to maintain the probe resistance, and hence the surface temperature (θ_s) constant. If the temperature and velocity of the water entering the sensor are also held constant, the last term of Eq. (1) has a fixed value. Changes in heat transfer between the sensor and the environment are then directly reflected by changes in power input from the compensating circuitry.

Since the heat transferred between the probe surface and the environment is the variable measured, environment temperature (θ_e) must be ob-

* This work was sponsored in part by Project Squid which is supported by the Office of Naval Research, Department of the Navy, under contract Nonr 1858(25) NR-098-038. Reproduction in full or in part is permitted for any use of the United States Government.

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tained from the term [Eq. (1)]:

$$h_0 S_0 (\theta_s - \theta_e)$$

The heat transfer coefficient h_0 is dependent on temperature, composition, velocity, and density. Ideally, to measure temperature, all other variables should be removed. Although this ideal has not been accomplished, it is possible to make h_0 largely independent of environment velocity by mounting the sensor in the throat of a sonic orifice. The velocity past the probe is then dependent on gas properties only.

An aspirating type probe with a sonic orifice ($M = 1$) is shown in Fig. 2. Rather than mounting the probe across the orifice, it is mounted in an enlarged section just upstream. This permits a smaller orifice (to reduce sample size) and decreases heat transfer to the sensor. The only requirement is that the velocity past the sensor must be higher than the environment velocity so all gas passing the sensor is sucked into the sonic orifice.

The probe tip represents a design problem

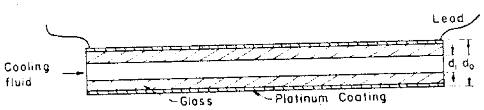


FIG. 1. Schematic diagram of sensing element.

where a compromise must be made. Since the approach flow is distorted as it is sucked into the probe, it is desirable to limit the sensitive area to the center portion of the probe. This can be done by electroplating a relatively thick metal layer along the portions of the element not to be used as a sensor. The difficulty with this design is the tendency for the cooling water to heat up before it reaches the sensitive portion. This causes a signal drift when the environment temperature changes rapidly.

The area of the sonic orifice represents another compromise in design. To minimize the sampling area, it is desirable to have the velocity past the sensor approximately equal to the approach velocity of the stream. To completely remove velocity fluctuations in the external fluid, the velocity past the sensor should be high compared to the stream velocity. Clearly a compromise must be made. It may be possible to mount the probe in the supersonic stream behind the orifice to help remove velocity effects of the environment. In this position, the probe is also well shielded from radiation effects and heat transfer coefficients are most easily correlated.

Compensating Circuitry

The compensating circuitry consists of a Wheatstone bridge and amplifier system (Fig. 3)

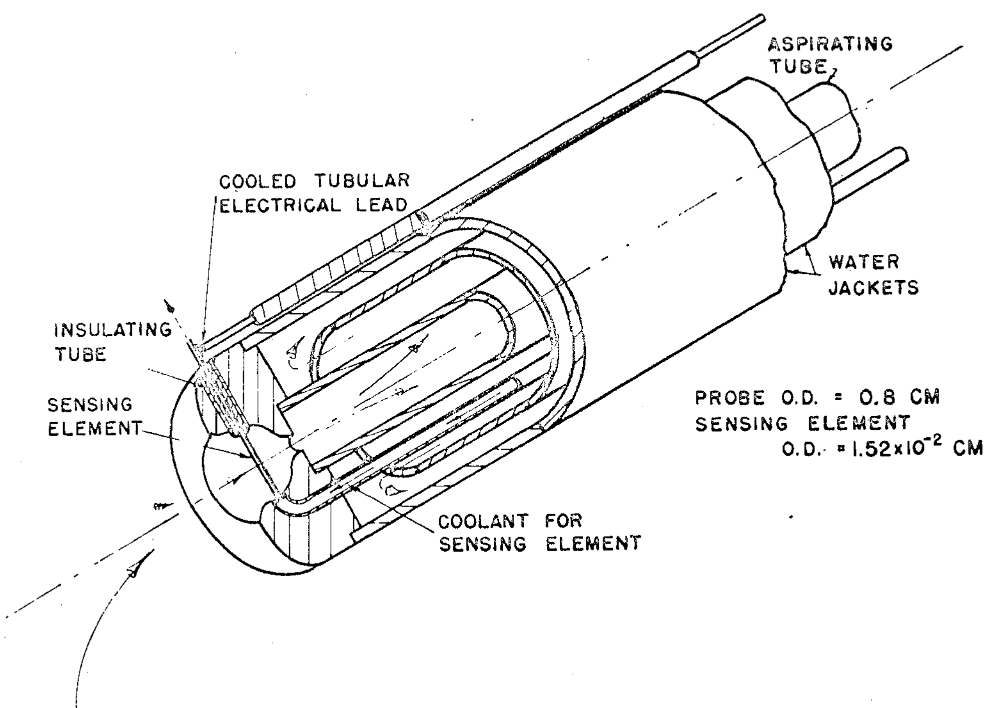


FIG. 2. Aspirating type probe with sonic orifice.

The platinum film on the probe is connected as one leg of the bridge. The bridge off-balance, or error signal, is amplified and the output current supplied to the top of the bridge as a correcting signal. An increase in probe resistance (temperature) will decrease the correcting current with a decrease in probe resistance having the opposite effect. The output of the system is the voltage across the bridge. A simple computing circuit can be added to give a direct reading of power supplied to the probe. The probe resistance, and hence surface temperature, is adjusted with the variable resistor R_1.

The circuit of Fig. 3 is a modification of the differential amplifier normally used for constant temperature anemometers. The required high gain, wide bandwidth, high power output, and high rejection ratio for in-phase signals was beyond the capabilities of available amplifiers. To overcome this problem, a dual amplifier was built with one amplifier "floating" on the bridge. The floating amplifier G_v has a voltage gain, while the amplifier feeding the top of the bridge, G_I, is a current amplifier. This system has proven quite satisfactory.

STEADY-STATE PARAMETERS

Although the probe can be used in measuring both steady-state and transient stream properties, some of the parameters upon which the interpretation of heat flux depends can be discussed more conveniently as steady-state effects. These concern themselves primarily with the problems of relating the measured heat flux to the properties of the approaching stream in applications where the probe cannot be calibrated over the operating range. It is usual to first consider the heat flux by convection to an idealized infinite cylinder and then correct for nonidealized realities which are referred to as sources of errors.

Nusselt Number

An account of the relationship between stream parameters and the heat flux to small wires has been well reported by Baldwin.[1] Herein it is seen that the Nusselt number for heat transfer to a cylinder transverse to a flowing stream is given by

$$\mathrm{Nu}(\mathrm{Pr}, \mathrm{Re}, M)$$

where Pr represents fluid properties, Re represents the influence of velocity, and Mach number is a way of introducing mean-free-path effects.

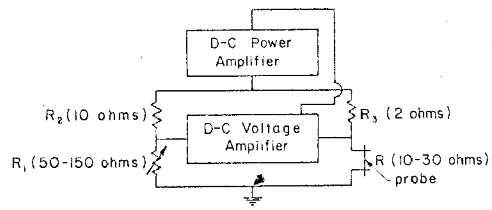


FIG. 3. Schematic of the bridge and amplifier layout for the compensating circuitry.

When the cylinder is placed inside a choked aspirating probe such that the Mach number at the probe is fixed, the velocity is eliminated as an independent variable and Nusselt number, for fixed Mach number, is dependent on the composition, pressure, and temperature of the gas. The relationship that has been added is the dependence of sound speed on fluid properties and temperature. Thus, although the direct effect of approach flow velocity can be eliminated by coupling velocity to fluid properties, it is still convenient to employ the conventional consideration of the flow past an isolated cylinder. The prospect of being able to calculate external temperature by measuring the heat flux to a cylinder of known surface temperature is, briefly, the following.

The report by Baldwin allows this calculation if fluid properties are nearly those of air, if $10^{-2} < \mathrm{Re} < 10^6$ and if $0 < M < 6$. The data collected by Baldwin indicates that his correlations are valid up to air temperatures of 1700°C. At temperatures where dissociation effects occur, the dimensionless correlation will have to be modified.

Rossner[2] discusses the effects of chemical dissociation and recombination near the boundary of a cooled surface.

These effects clearly can influence the heat transfer to the cylinder when it is operated in a dissociated gas. In general, this problem is quite complex, involving the kinetics of the recombination process. There are some limiting conditions, however, that permit determination of heat transfer rates.

If the recombination time is long compared to the time it takes the particle to diffuse through the boundary layer,

(1) the heat transfer for frozen composition can be used if no reactions occur on the probe surface, or
(2) the heat transfer is the sum of the values compiled in (1) plus the energy transported to the surface by diffusion of species that recombine exothermically on the surface.

If the recombination time is short compared to the time it takes a particle to diffuse through the boundary layer, then the gas can be assumed to be in chemical equilibrium and the method of Brokaw[3] can be applied.

It is not clear at this time under what conditions the above limiting treatments are applicable. Further, the above methods have not been verified experimentally for cylinders. It is clear, then, that proper interpretation of heat flux to the cooled cylinder at temperatures where appreciable dissociation occurs awaits further work.

Radiation Errors

When required, the sensing element can be arranged such that it "sees" only the water-cooled walls of the probe. Unless an oxide forms that permits the walls to heat up above 500°C, the radiation error is easily shown to be negligible.

Effect of Heat Transfer to Nozzle Wall

Simmons and Glawe[4] have shown that if care is taken in the matching of probe orifice diameter to operating pressure, probes of convenient size can be employed such that the discharge coefficient, hence the Mach number at the sensing element, is nearly independent of the temperature difference between wall and flowing stream. Attention must also be given to insure that the sensing element is kept out of the thermal boundary layer.

Conduction Losses to Probe Support

The low thermal conductivity of the glass tubes give a significant reduction in end losses when compared with solid metal wires. In terms of length to diameter ratio, reduction by a factor of 20 is possible for the same end losses. The present sensor has a conduction loss of about 5% when used in the aspirating type probe. These losses can be calculated quite easily, and can, therefore, be taken into consideration in steady-state measurements.

Nonuniformities in Surface Temperature

The surface temperature distribution of the probe can be nonuniform due to variations in either the external or the internal heat transfer coefficient. Certainly, the internal heat transfer coefficient will vary since the thermal boundary layer must build up as the cooling water enters the probe. This effect will be particularly prominent when the cooling fluid is in the laminar flow regime. An analysis of the effect of this on the heat transfer to the cooling fluid under various operating conditions has been carried out. With present probes, the maximum error is about 2 %. With a poorly designed probe, it is possible to get errors over 10%, so it is necessary to consider this aspect in probe design.

Extreme nonuniformities in external heat transfer along the probe can also cause rather large errors. With the small sensor considered here and the aspirating type probe system, this nonuniformity has not been considered serious.

DYNAMIC MEASUREMENTS

With transient conditions, two additional sources of output error occur. The primary one of concern here is the lag of the sensitive element which occurs even with compensation. The second source of lag is the time it takes for the boundary layer to establish itself around the probe. Sparrow[5] and Lighthill[6] have investigated this for the region near the stagnation point, but more work needs to be done. The primary concern here is the frequency response limitations of the sensitive element and compensating circuitry. The derivation of the response of the cooled cylinder and compensating circuitry has been given in a previous report. Only the essential results will be presented here.

A useful parameter in discussing the response characteristics of the probe is the thermal impedance. This is defined as:

$$Z = \frac{\theta}{q}$$

and for the cooled probe is:[7]

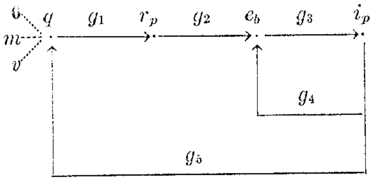
$$Z = \frac{1}{k_g S_0 \sqrt{\omega/\alpha}\,(A + Bi)} \tag{2}$$

where A and B depend on the probe diameter (inside and outside), probe cooling rate, and the parameter $a\sqrt{\alpha/\omega}$. The limiting frequencies are of primary interest:

$$Z = \frac{1}{US_0} \quad \text{as} \quad \omega \to 0$$
$$Z = \frac{1}{k_g S_0 \sqrt{\omega/2\alpha}\,(1 + i)} \quad \text{as} \quad \omega \to \infty \tag{3}$$

The complete equations for A and B are rather lengthy[7] and will not be presented here. Figure 4 gives the results of a complete calculation and also the approximation using only the limiting condition. The straight line values give minimum conditions except for a very small region near the corner frequency.

The closed loop gain of the system can be derived using transfer functions. The signal flow graph[8] of the system in Fig. 3 is:



where:

$$g_1 = \frac{\lambda R_r}{(1/Z) + h_0 S_0 - P\lambda (R_r/R_p)} \quad \text{(Appendix II)}$$

$$g_2 = -I_p \frac{R_3}{R_p + R_3}$$

$$g_3 = \frac{G}{R_3 + (1 - G_I)R}$$

$$g_4 = \frac{R_3 + (1 - G_I)R}{G}\left[1 - \frac{K}{E_T}\right]$$

$$g_5 = 2\frac{P}{I_p}$$

Writing the equation for the closed loop gain gives:

$$G_{CL} = \frac{-1}{\left\{\frac{[(1/Z) + h_0S_0 - P\lambda(R_r/R_p)][R_3 + (1 - G_I)R][R + R_3]K}{2P\lambda R_r G R_3 E_T}\right\} + 1} \qquad (4)$$

For perfect response, G_{CL} should equal minus 1. The dependence of the frequency response on the power input from the circuit in the terms P and E_T is particularly important for measurements of high temperatures. As heat transfer to the probe from the external environment increases, the power from the circuit must decrease [Eq. (4)] and the frequency response of the system also decreases. Figure 5 shows the relation between maximum frequency response and heat transfer between probe and environment for the parameters indicated.

The sources of error present in steady-state measurements are also present under dynamic conditions and may be more of a problem. Any shift in probe conditions due to varying heat fluxes from the environment can show up as a slow drift of the signal superimposed on the output. To determine the actual response, a setup was devised to give a nearly step change in temperature at constant pressure.

EXPERIMENTAL RESULTS

The experimental results reported here were obtained by using a "flame kernel" to give a nearly square wave in temperature. This test is effective in determining the possible time lags in the system as well as the tolerance of the probe to a high-temperature gas.

The flame kernel is generated by igniting a combustible mixture as it flows from a nozzle. The nozzle is 5 centimeters in diameter, the combustible mixture is propane air in required amounts; ignition is accomplished by a spark across a pair of streamlined electrodes placed in the stream.

The thin flame front enveloping the kernel is laminar, and contains hot exhaust products at flame temperature. The temperature distribution has been measured by a sodium D-line[9] reversal technique and found to be flat across the kernel.

The flame front is thin compared to the diameter of the kernel such that when it is carried past the probe by the flowing combustible mixture, there results at the plane of measurement a very nearly square temperature wave. The shape of the square wave can be adjusted by changing fuel air ratio, spark frequency, and mixture flow rate.

Figure 6 shows the results of an experiment with the aspirating probe. The response time is seen to be greater than that predicted theoretically. Some of the time lags expected from purely physical considerations are:

(1) time for flame front to pass probe $\cong$ 70 microseconds

(2) time to pass from probe to orifice $\cong$ 80 microseconds.

Effects which can cause additional lags in the system are:

(1) distortion of approach flow at the sensor and at the orifice[10]

(2) the time required for the thermal boundary layer to build up around the probe.

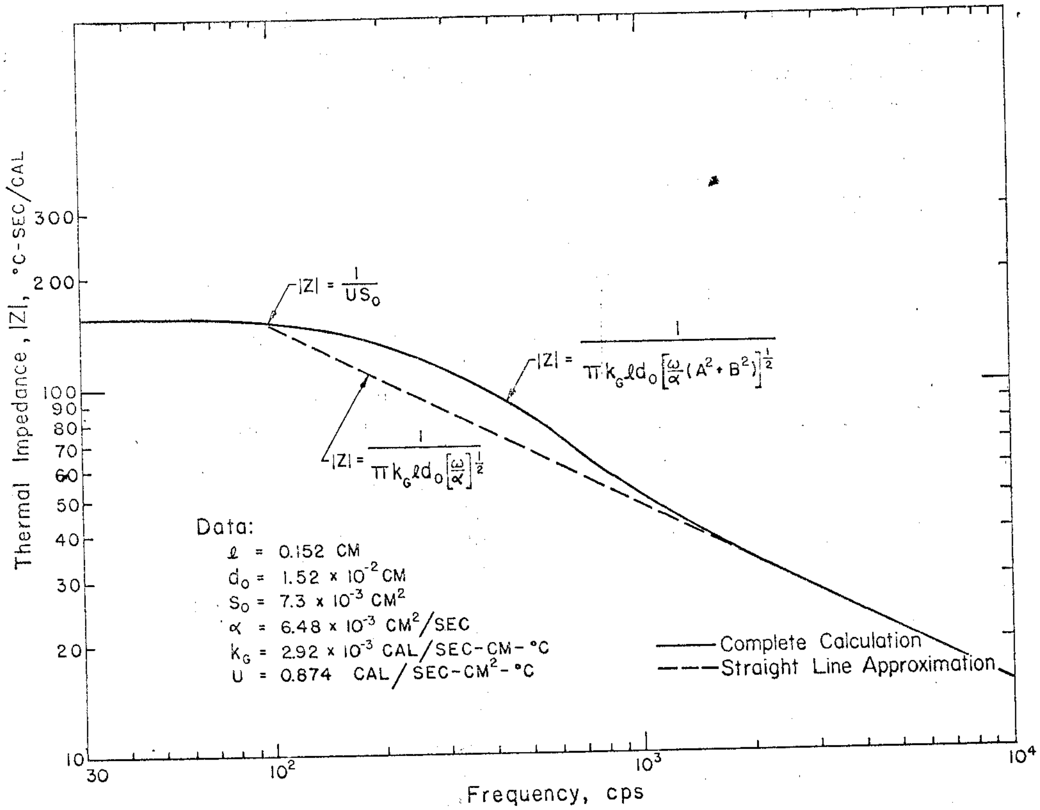


FIG. 4. Thermal impedance Z as a function of frequency.

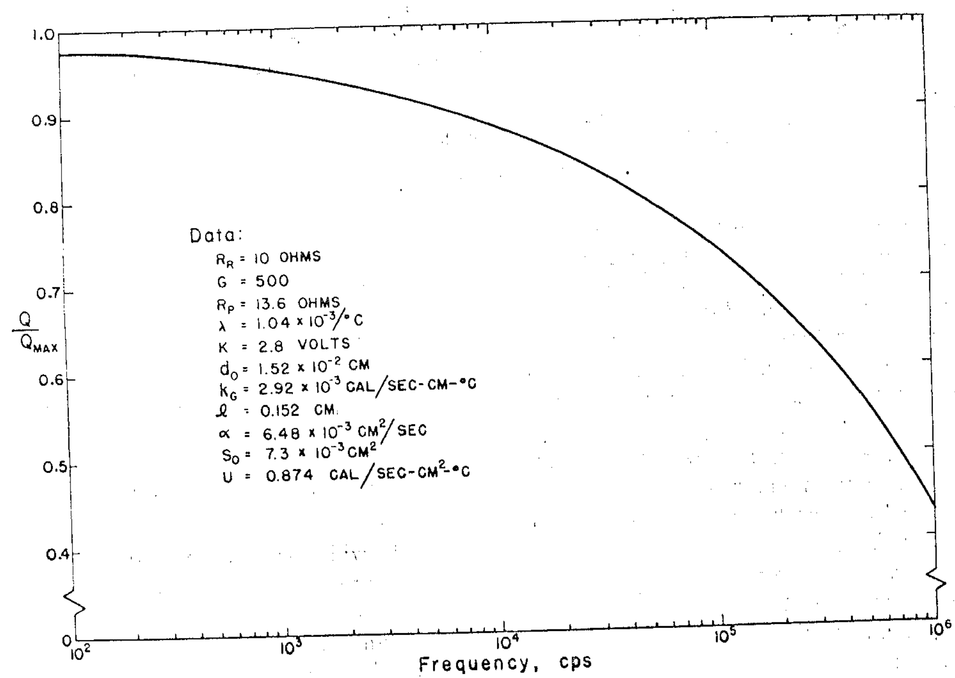


FIG. 5. Dependence of frequency response (within 1 decibel) on the heat transfer Q from environment to probe. Q_{max} = 301 cal/cm²-sec.

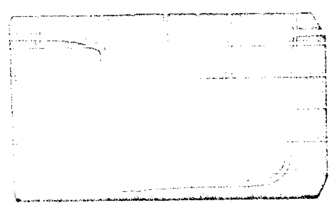


Fig. 6. Response of aspirating type probe to a flame kernel.

The slow drift of the signal while the probe is in the hot gas can be attributed to a change in probe water supply temperature.

CONCLUSIONS

A heat flux probe has been designed which is capable of withstanding a high temperature environment. Changes in heat transfer between the probe and the environment can be determined directly from the changes in power supplied by the compensating circuitry. The maximum heat flux is limited by the cooling rate in the sensor, the present design being capable of heat fluxes on the order of 300 cal/cm²-sec.

To help remove effects of environment velocity on the probe output, a sonic aspirating nozzle is used with the probe. With this system, velocity past the probe depends on gas properties. With the present design, boundary layer effects and other time delays slow up the probe response below that theoretically expected.

The aspirating heat flux probe in its present form has a response time on the order of 250 microseconds and is capable of withstanding gas temperatures greater than 3000°C at atmospheric pressure. Accuracy is primarily dependent on calibration or the validity of the heat transfer relations when calibration is not possible. It is believed that response time and maximum heat flux can be increased with further design improvements.

APPENDIX I

Nomenclature

d_0 = Outside diameter of probe.
E_T = Voltage on bridge.
g = Transfer function (specific functions with subscripts defined in text).
G = Total gain of the compensating amplifiers (not including bootstrap effect).
G_I = Gain of current amplifier.
G_v = Gain of the "floating" voltage amplifier.
G_{CL} = Closed loop gain of the entire system.
h_0 = Heat transfer coefficient between environment and probe surface.
I_p = Current in probe.
K = Voltage on the bridge when the bridge is at balance (no input voltage to compensating circuit).
k_g = Thermal conductivity of probe material.
k_f = Thermal conductivity of environment fluid.
Nu = Nusselt number = $h_0 d_0 / k_f$
M = Mach number = $(\gamma R \theta_e)^{1/2}$ (temperature in °K)
m = Fluctuations in molecular weight.
P = Electrical power input to probe.
Pr = Prandtl number = ν/α
q = Small change in heat transfer from probe surface.
Re = Reynolds number = $V d_0/\nu$
R_p = Resistance of probe.
R_r = Resistance of probe at 0°C.
R_3 = Resistance of fixed bridge resistor in series with probe.
S_0 = Surface area of probe.
U = Heat transfer coefficient from probe surface to cooling water based on probe surface area.
V = Free stream velocity of environment.
Z = Thermal impedance of probe defined by $Z = (\theta/q)$.
α = Thermal diffusivity.
θ = Temperature.
θ = Small change in temperature.
λ = Temperature coefficient of resistance.
γ = Ratio of specific heats of environment fluid
ω = Radial frequency.

Note: Small letters not defined above indicate a small change in the variable corresponding to the identical capital letter.

Subscripts

e = Mean property of environment.
s = Value of property of probe surface.
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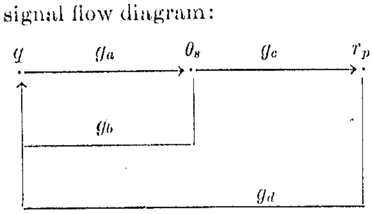
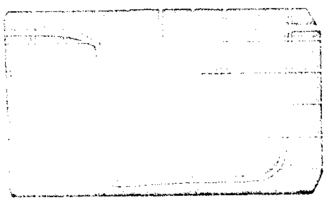
w = Mean property values in coolant.

APPENDIX II

Derivation of Transfer Function for the Closed-Loop Gain Equation

The transfer function g_1 is derived from the following signal flow diagram:





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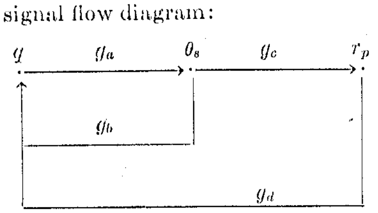

The function g_a has been defined in the text as the thermal impedance Z. The function g_b represents a change in heat transfer between the probe surface and the environment due to a change in probe surface temperature. Since:

$$Q = -h_0 S_0(\theta_s - \theta_e)$$

$$g_b = \frac{q}{\theta_s} = -h_0 S_0$$

To calculate g_c, the probe resistance is assumed to be a linear function of temperature over the range considered. Then:

$$R_p = R_r(1 + \lambda\theta_s)$$

and

$$g_c = \frac{r_p}{\theta_s} = R_r\lambda$$

Finally, the electric power input to the probe is:

$$Q = I_p^2 R_p$$

$$g_d = \frac{q}{r_p} = I_p^2 = \frac{P}{R_p} \qquad (1)$$

The equation for the above signal flow graph[8] is:

$$g_1 = \frac{r_p}{q} = \frac{g_a g_c}{1 - g_a g_b - g_a g_b g_c}$$

Substituting into the above equation gives:

$$g_1 = \frac{\lambda R_r}{(1/Z) + h_0 S_0 - \lambda P(R_r/R_p)}$$

This is the equation given in the text.

To obtain the closed loop response equation, the functions g_2, g_3, g_4, and g_5 must be determined. The off-balance voltage of the bridge can be written as follows:

$$E_b = \frac{R_1 R_3 - R_2 R}{R_1 + R_2} I_p \qquad (2)$$

In this equation, an assumption has been made about the return current to the floating amplifier which must pass through R_2. This current must either be very small, so it can be neglected, or it must be a linear function of the system output current. In the latter case, the only effect is to make the resistance R_2 appear slightly larger than its actual value by the factor:

$$I'R_2/I_1$$

where: I' = return current to floating amplifier

I_1 = current in resistor R_1.

If the above cases do not exist, the return current must be considered as another feedback term. The transfer function g_2 is now, from Eq. (2):

$$g_2 = \frac{e_b}{r_p} = \frac{-I_p R_2}{R_1 + R_2} \approx -I_p \frac{R_3}{R_p + R_3}$$

The value of g_3 and g_4 must be obtained by considering the amplifier system. If we let the voltage on the bridge E_T equal K when the bridge is balanced ($E_b = 0$), then

$$E_T = K + E_b G_T \qquad (3)$$

$$G_T = \frac{G}{1 - \dfrac{G_I R}{R + R_3}} \qquad (4)$$

where: G_T = total voltage gain of system

G = total voltage gain of the two amplifiers.

Combining Eqs. (3) and (4):

$$\frac{E_b}{I_p} = \frac{R_3 + (1 - G_I)R}{G}\left[1 - \frac{K}{E_T}\right]$$

For linear amplifiers:

$$g_4 = \frac{e_b}{i_p} = \frac{R_3 + (1 - G_I)R}{G}\left[1 - \frac{K}{E_T}\right]$$

From Fig. 3, the factor g_3 has the following value:

$$g_3 = \frac{i_p}{e_b} = \frac{G}{R_3 + (1 - G_I)R}$$

The feedback to the probe from the output is the transfer function g_5. From Eq. (1),

$$g_5 = q/i_p = 2I_p R_p = 2P/I_p$$

This completes the derivation of the transfer functions used in Eq. (4) of the text.

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