TECHNICAL BULLETIN

Bulletin No. 2

BASIC PERFORMANCE DATA FOR HEAT FLUX SYSTEM

The attached material is designed to give you additional technical data on the HEAT FLUX SYSTEM.

We hope that this information will help you in determining the applicability of the HEAT FLUX SYSTEM in solving measurement problems in your area. Any questions you may have will be given careful consideration. Please address your inquiries on technical questions to Dr. L. M. Fingerson, Director of Research and Development.

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BASIC PERFORMANCE DATA FOR HEAT FLUX SYSTEM

Summary

The Heat Flux System is basically a constant temperature compensated anemometer capable of operation in high temperature gases. This increased operating range is obtained with an internally cooled sensor which permits making measurements with the sensor surface temperature below the environment temperature. With the entire probe structure water cooled various configurations for interpreting heat transfer in terms of temperature, velocity, or molecular weight have been devised for use in hot gases.

Equations and Figures for both the per cent error and signal to noise ratio as a function of frequency are presented. The ordinary hot wire is used as a comparison in determining characteristics of the film type probes (platinum film on glass substrate) generally used with the Heat Flux System (Figure 1). This comparison has significance in low temperature environments only since the hot wire cannot be cooled. However, it can be seen that the necessity of using film probes in high temperature environments does not sacrifice operating characteristics, but actually results in an improvement over hot wires of similar dimensions.

In addition to comparative data between hot wires and hot film sensors, performance data on the internally cooled sensors is also included. It is the cooled sensors that constitute the really unique feature of the Heat Flux System. These sensors can be operated beyond the temperature range of both thermocouples and hot wires.

The Heat Flux System has been designed for exceptional ease of operation. A complete reviewal of this paper is certainly not required for successfully applying the Heat Flux System to fluid flow measurements. However, a number of requests have been received for more detailed performance data and it is hoped that this paper will provide the required information.

BASIC PERFORMANCE DATA FOR HEAT FLUX SYSTEM

A complete analysis of the Heat Flux System is given in reference [1]. The derivation of equations will not be given in this brief report but rather some observations from the results. In addition to performance data on per cent error and signal to noise ratio, data on end losses and information on the compensating circuit are also given. Before going to the dynamic characteristics of the Heat Flux System, a steady state analysis of its operation is presented to clarify its basic mode of operation.

Operating Principle - Steady State Analysis

The constant temperature compensated hot wire or hot film anemometer measures the rate of heat transfer between the environment and the sensor which is held at constant temperature by a feedback circuit. A heat balance for the electrically heated hot film (or hot wire) can be written as follows:

Electrical Power to Film = Heat Loss to Outside Environment + Heat Loss to Interior of Sensor

For steady state conditions, the only heat loss to the interior for the hot film (or hot wire) sensor is due to conduction to the supporting structure ("end losses" for cylindrical sensors). If this loss is neglected the heat balance can be written:

$$P = h_o S_o (t_s - t_e) \qquad (1)$$

where

P = Electrical power to sensor from compensating circuit, BTU/hr

h_o = Heat transfer coefficient between sensor and environment, BTU/hr-ft^2-^{o}F

S_o = Area of controlled, heated surface of sensor, ft^2

t_e = Temperature of environment, ^{o}F

t_s = Temperature of sensor, ^{o}F

Since electrical power, P, must always be positive, equation (1) is satisfied only when the film temperature, t_s, is greater than environment temperature, t_e.

If a heat sink is added to the interior of the sensor the limitation of $t_s > t_e$ can be removed. In the Heat Flux System cooling water is used as a heat sink in a tubular element and the heat balance becomes:

$$P = h_o S_o (t_s - t_e) + U(t_s - t_w) \qquad (2)$$

where

U = Overall heat transfer coefficient between sensor surface and cooling water, (based on average water temperature) BTU/hr-°F

t_w = Average temperature of cooling water, °F

When this sensor is operated with a constant temperature circuit, the film temperature, t_s, is held constant. With the entering water temperature, t_w, also held constant the last term of equation (3) is:

$$U(t_s - t_w) = C \qquad (3)$$

where

C = Constant heat transfer rate from sensor surface to cooling water, BTU/hr

The heat transfer between surface and environment is positive if $t_s > t_e$ and negative when $t_s < t_e$. The maximum heat flux that can be withstood while maintaining t_s constant is found by setting electrical power, P, equal to zero. Then:

$$h_o S_o (t_s - t_e) + C = 0 \qquad (4)$$

The maximum environment temperature from (4) is:

$$t_e = t_s + C/h_o S_o$$

The constant, C, can be obtained experimentally by reducing $h_o S_o (t_s - t_e)$ to a negligible quantity by shielding the probe from the environment. Under these conditions, from (2) and (3),

$$P = C.$$

Since the circuit has a direct power read-out, the value of C in watts is merely read off the meter.

Characteristic Equation for System Response

The Heat Flux System, as with a compensated hot wire anemometer, has a frequency response in the kilocycle range. The exact value of the response for a given situation depends on the circuit characteristics, environment conditions, and sensor type. It should be pointed out that varying the conditions affects only the maximum response. This is unlike the constant current system where the entire compensated frequency range must be reset after changing mean environment conditions or sensors. In fact, the constant current system cannot give accurate results for large fluctuations about the mean. The Heat Flux System, as with other constant temperature systems, always has the correct compensation for all conditions within its operating limits and frequency response.

The basic arrangement of a constant temperature system is shown in Figure 2. This "closed loop" system maintains the sensor resistance, R_p, and hence sensor surface temperature, t_s, nearly constant by continually balancing the bridge through the amplifier. How accurately the bridge is maintained at balance is a measure of the system's operating characteristics. A more meaningful measure, however, is concerned with how well the changes in electrical power to the bridge follow the changes in heat flux between the sensor and its environment.

The frequency response will be presented in terms of per cent error in amplitude. This can be defined as follows:

> Per cent Error = (|Change in Heat Transfer Between Sensor and its Environment, Watts| − |Change in Electrical Power to the Sensor, Watts| ÷ |Change in Heat Transfer Between Sensor and its Environment, Watts|) x 100.

To obtain a per cent error versus frequency curve, a sinusoidal change in heat transfer between sensor and environment is assumed. As the frequency of this change increases, it becomes increasingly difficult for the sensor to follow, and the amount of compensation required of the circuit increases.

The equation for per cent error, from reference [1], can be expressed as follows:

$$E = \left[1 - \frac{1}{\dfrac{\left[\frac{1}{Z} + h_o S_o - P \lambda R_r / R_p\right] (R_p + R_3)^2}{2 P \lambda R_r G R_3} \left(\dfrac{X}{E_T}\right) + 1} \right] \cdot [100] \qquad (5)$$

where:

Z = Thermal impedance, $\frac{{}^{\circ}F - hr}{BTU}$

λ = Temperature coefficient of resistance, $^{\circ}F$

R_r = Reference resistance of sensor, ohms

R_p = Sensor resistance at operating conditions, ohms

R_3 = Resistance of bridge resistor in series with sensor, ohms

G = Gain (voltage amplification) of circuit

E_T = Voltage on bridge under operating conditions, volts

E = Voltage on bridge when bridge is balanced, volts

The term of (5) most complex and unfamiliar to most is the one designated Z, thermal impedance. This quantity will be discussed at some length since it is a characteristic of the sensor itself.

Thermal Impedance

The parameter which characterizes the sensor has been designated an impedance because of its similarity to electrical impedance. The thermal impedance is defined as:

$$Z = \frac{t_s'}{q} \qquad (6)$$

where:

t_s' = Change in characteristic sensor temperature $^{\circ}F$

q = Change in heat transfer to sensor, BTU/hr

Comparing thermal to electrical quantities in equation (6), temperature, t_s', is the driving force rather than voltage and heat, q, is transferred rather than current. The sensors can have both thermal resistance and capacity. (There is no thermal term equivalent to electrical inductance.)

The thermal impedance for film probes, from 1 is:

$$Z = \frac{1}{k_g S_o \sqrt{\frac{\omega}{\alpha}} (A + Bi)} \tag{7}$$

where

k = Thermal conductivity of glass, BTU/hr-ft-°F

ω = Circular frequency, hr^{-1}

α = Thermal diffusivity = $\frac{k_g}{\rho_g C_{p_g}}$, ft^2/hr

ρ_g = Density of glass, lbs/ft^3

C_{p_g} = Specific heat of glass, BTU/lb-°F

A, B = Factors depending on the parameter $a\sqrt{\frac{\omega}{\alpha}}$

a = Probe radius, ft

Note: Thermal effect of platinum film is negligible as shown in [1].

The factors A and B are plotted in Figure 3. A general curve is shown for the film sensors with a solid glass rod as the supporting structure. For the cooled tube sensors, the curve depends on the ratio k_g/h_i and the value of both $a\sqrt{\frac{\omega}{\alpha}}$ and $b\sqrt{\frac{\omega}{\alpha}}$. Therefore, a general curve cannot be drawn. It can be shown, however, that the value of Z for cooled probes can be closely approximated by the two curves:

Low frequencies

$$Z = \frac{1}{U} \tag{8}$$

High frequencies

$$Z = \frac{1}{k_g S_o \sqrt{\frac{\omega}{2\alpha}} (1 + i)} \tag{9}$$

Equation (9) is the thermal impedance for a semi-infinite wall. All film probes should approach (9) at high frequencies where the temperature oscillations can penetrate the surface only a very small distance. As indicated by (8), at very low frequencies the entire glass tube follows the temperature oscillation and the thermal impedance is just the resistance to heat transfer from the probe surface to the cooling fluid. The curves of equation (8) and (9) "meet" at about 100 cylces per second for the 0.006 in. O.D. by 0.004 in. I.D. tubular sensors.

Hot wires can be represented quite accurately by a simple thermal capacity as follows:

$$Z = \frac{1}{i\omega C_T} = \frac{4}{i\omega\rho_w C_{p_w} d_o S_o} \qquad (10)$$

where

C_T = Thermal capacity of wire, BTU/°F

ρ_w = Density of wire, lbs/ft^3

C_{p_w} = Specific heat of wire, BTU/lb-°F

d_o = Wire diameter, ft

A little reflection will show that the above equation is inaccurate unless the wire has infinite thermal conductivity so the cross-section is at uniform temperature for any frequency. It has been shown, however, [2] that the error caused by ignoring the temperature nonuniformities in a wire is small, and that equation (10) is satisfactory.

With equations (7) and (10) one can use equation (5) to calculate the per cent error for systems using either film probes or wires.

Response of Typical Sensors

Results of calculations using equation (5) are given in Figures 4 and 5. Parameters entering the equation are given in Table I. Actual Heat Flux System values were used for the bridge resistors and circuit gain. Sensor properties were obtained from standard references with the exception of the thermal conductivity of glass, k_g. An experimental check of equation (5) in reference [1] showed that k_g was abnormally high for the thin-walled glass sensors. Therefore the value derived from the results in [1] is used.

The exact meaning of the per cent error versus frequency curves can best be explained by an example. Assume a sinusoidal fluctuation in heat transfer between the sensor and its environment of one BTU/hr. If the amplitude of the electrical power (correcting signal) to the sensor under these conditions is 0.98 BTU/hr, then the error is 2 per cent. The sinusoidal change in heat transfer could be generated by a periodic variation in temperature, velocity, or composition. Phase shift is not shown since it is under 10° for film sensors when the error is less than 30 per cent.

The rapid increase in error at high frequencies for the wire, when compared with the film sensor, is evident in Figure 4. This superiority shown by the film sensor is due to the difference in location of the resistance element which is sensed and controlled by the circuit. In the wire, current is conducted through the entire cross-section and the circuit responds to the average resistance value of this cross-section. With film sensors, the current is conducted only in the surface film and the circuit responds to average resistance of this film. It is apparent that environment changes will have the most influence on the exterior surface of the sensor.

Using 10,000 cycles per second as an example, the per cent loss in electrical signal when compared with the heat flux change for the sensors in Figure 4 is:

Sensor	Per Cent Error
0.006 in. Wire (Pt)	84.4 (Off Figure)
0.006 in. Film (Uncooled)	21.1
0.006 in. Film (Cooled)	7.3
0.001 in. Wire (Pt)	12.1
0.001 in. Film (Uncooled)	10.4
0.0002 in. Wire (Pt)	0.5

The effect of electrical power to the sensor, P, clearly shows up when the sensor is cooled. The 0.006 in. cooled sensor has better response at this frequency than either of the 0.001 in. sensors.

A difficulty in making direct comparisons between film sensors and wires is the large difference in electrical resistance, R_p, for a given diameter and length. To remove this factor the curves of Figure 4 were calculated with realistic values of size and resistance for film sensors and then these same values were used for the wires. This choice is actually optimistic for the wires since one of comparative size would have a lower resistance than the film sensor.

Results for cooled probes are given in different form on Figure 5. Since these probes are all the same size (0.006 in. O.D. by 0.004 in. I.D.) a generalized plot to include the effect of electrical power to the probe is presented.

The film probes can be built in a variety of shapes and sizes and the 0.006 in. tube and 0.001 in. solid rod presented here should certainly not be considered as limiting forms. They are used because they do seem to have wide application and are the sizes readily available at present.

Compensating Circuit Details

The circuit shown in Figure 2(a) is the standard arrangement for constant temperature compensating equipment. The Heat Flux System, shown in Figure 2(b), has several modifications which give it some distinct advantages in flexibility, performance, and ease of operation.

A major modification is the floating amplifier, G_V, shown in Figure 2(b). This arrangement gives the system very good stability since the carefully matched inputs of the straight differential amplifier are not necessary. With this system the value of G in equation (5) is:

$$G = \frac{G_I G_V (R_p + R_3)}{R_3 + (1 - G_I) R_p} \qquad (11)$$

The current amplifier, G_I, is an emitter-follower configuration with a voltage gain of less than one under all operating conditions. From the values in Table I, G is 1350 for the high sensitivity bridge ($R_3 = 40$) and 5,000 for the high power bridge ($R_3 = 2$). Changes in the ratio of $R_3/(R_p + R_3)^2$ in equation (5) reduces the apparent difference in amplification so the bridge selected has a very small influence on the frequency response calculations. The primary influence of the two bridges is on power dissipation and signal to noise ratio. This will be discussed in the next section of this report.

The Heat Flux System is provided with controls to balance the bridge for the mean environment conditions. These same controls can be used to actually adjust the effective gain of the circuit and thus decrease the errors shown in Figures 4 and 5. This is a result of the "regenerative" feedback which occurs when the bridge is unbalanced in one direction. The off-balance of the bridge, E_b, therefore influences frequency response.

Bridge off-balance does not occur directly in equation (5) but shows up indirectly in the ratio K/E_t. The relation between E_b, E_t, and K is:

$$E_t = K + E_b G$$

where

E_b = Bridge Off-Balance

K = Bridge Voltage When Bridge is Balanced ($E_b = 0$)

E_t = Actual Bridge Voltage

From equation (5) it can be seen that a small value of K will decrease per cent error for a given frequency.

The previous error curves of Figure 4 were presented for a "balanced" bridge. The stability of the amplifier system of Figure 6 permits increasing the effective gain by setting $K < E_T$. In many cases the gain can be increased a factor of five or more by utilizing this "regenerative feedback" effect. The curves of Figure 5 were plotted with K = 4 volts. This is a realistic value and, when measuring large fluctuations, K must be constant while permitting K/E_T to vary as E_T changes.

The curves of Figures 4 and 5 were plotted to 100 KC. The present amplifier can be considered stable up to about 20 KC with the adjustable bridge. It is expected that this range will soon be extended with an external, fixed resistor bridge that can be obtained as an accessory and plugged into the present amplifier system.

A bridge system is very sensitive to external lead reactance and other possible modifications in the system. An adjustable filter on the feedback loop permits tuning the system for maximum frequency response for the given operating conditions. This filter also permits adjusting the loop so careful matching of external leads is unnecessary when maximum frequency response is not required. Also, under these conditions, per cent error can be reduced to a minimum by setting $K \ll R_M$.

Sensitivity

The signal to noise ratio, or sensitivity, is a very important factor in a fast response instrument. The signal to noise ratio is essentially a parameter which indicates how small a change in the variable of interest can be detected or accurately measured. High frequency response, although desirable, has little value in turbulence measurements, for example, if the fluctuations cannot be separated from the background noise of the electronics.

The output noise in a closed loop system is not simply the input noise multiplied by the amplifier gain, as in an open loop system. Rather, the output noise is also affected by the sensor being used and other circuit parameters as shown in the following equation from [1].

$$\frac{e_t}{e_{tn}} = \frac{0.541\ (q/e_{bn})\ P^{\frac{1}{2}} \lambda R_r}{R_p^{\frac{1}{2}} (1 + R_p/R_3)\left[\frac{1}{Z} + h_o S_o - P \lambda R_r/R_p\right]} \qquad (12)$$

where

e_{bn} = Input noise of amplifier, volts

q = Input signal, BTU/hr

e_t = Output signal of system, volts

e_{tn} = Output noise of system, volts

It is interesting to note that the amplifier gain does not even appear in equation (12). A factor which does show up is the $(1 + R_p/R_3)$ in the denominator. This term has the value 1.5 for the high sensitivity bridge and 11 for the high power bridge. The small value of R_3 in the high power bridge is to keep the power dissipation low where, in general, the high signal to noise ratio is not required.

Equation (12) is plotted in Figure 6 for the same sensors previously discussed. The calculations are based on the data given in Table II. The curves give the signal to noise ratio versus frequency for a change in heat flux of two per cent of the mean. Since the primary purpose of these curves is to compare sensors, a constant value of e_{on} (20×10^{-6} volts) was assumed even though in actual practice it will vary with frequency. Circuit noise from frequency components beyond the range of interest can be removed from the output with filters.

For probes of a given dimension, the films are again superior to the wires. Using 10,000 cycles per second as an example, the following values can be taken from Figure 6.

Sensor	e_t/e_{tn}
0.006 in. Wire (Pt)	0.430
0.006 in. Film (Uncooled)	7.48
0.006 in. Film (Cooled)	1.90
0.001 in. Wire (Pt)	2.40
0.001 in. Film (Uncooled)	7.35
0.0002 in. Wire (Pt)	36.6

Surprisingly, at 10,000 cycles per second the 0.006 in. film sensor is actually better than the smaller 0.001 in. film sensor. The large decrease in e_t / e_{tr} for the cooled sensor is due to changing to the high power bridge. The maximum output of the high sensitivity bridge is 1.5 Watts to the sensor, while the cooled sensor in this example required 5 Watts. The comments made regarding sensor size and resistance for Figure 4 are also valid for Figure 6.

Figure 7 shows the results with cooled tube sensors. Although several curves are given, a single curve can be generalized for any power level as shown. The assumed signal for these curves is two per cent of an 8 Watt range or 0.16 Watts.

Other Factors of Interest

One of the most valuable characteristics of film probes are their low end losses (conduction losses to the supports). Results of calculations given in reference [1] are as follows:

Probe Type	Environment		Probe Data				
	t_e	V	t_s	d_o (in.)	ℓ (in.)	ℓ/d	End Losses %
Wire (Pt)	70°F	500 ft/sec	800°F	3.10^{-4}	0.12	400.0	4.5
Film	70°F	500 ft/sec	800°F	3.10^{-3}	0.092	28.8	4.5
Cooled Film	70°F	500 ft/sec	400°F	6.10^{-3}	0.0636	10.6	4.5

In the above table the film sensor is actually shorter than the wire for the same end losses, even though the diameter is 10 times greater than that of the wire. This permits nearly point measurements with a more rugged probe when using film sensors.

Although the actual calculation is quite complex, the lower conductivity of the glass when compared to metal wires is largely responsible for the improvement.

Conclusions

Information on the frequency response, compensating circuit, signal to noise ratio, and end losses has emphasized the following points:

1) For a given diameter, film sensors have better frequency response and signal to noise ratio than hot wires.

2) In cases of low heat transfer rates between the sensor and its environment, internal cooling can improve both frequency response and signal to noise ratio.

3) The low end losses of film sensors permits a length to diameter ratio 15 times less than for hot wires. This is particularly valuable in three dimensional flows where nearly point measurements are desired.

4) The bridge balance controls and the exceptional stability of the floating amplifier permits increasing the effective gain of the Heat Flux System when desired.

5) An adjustable filter on the feedback loop can be used to adjust the Heat Flux System for maximum frequency response under given conditions or, alternatively, to set the loop for maximum stability when fast response is not required.

Finally, the internal cooling of the film sensor permits using the Heat Flux System in high temperature applications where conventional instruments will melt or vaporize when exposed directly to the environment.

The Heat Flux System has been designed for simplicity, consistent with the wide flexibility it has to offer. It will give valuable data in a wide variety of fluid flow systems, both liquids and gases.

<u>References</u>

1 Fingerson, L. M., "A Heat Flux Probe for Transient Measurements in High Temperature Gases," Ph.D. Thesis, University of Minnesota, 1961.

2 Lowell, Herman H., and Patton, Norman, "Response of Homogeneous and Two-Material Laminated Cylinders to Sinusoidal Environment Temperature Change, with Applications to Hot Wire Anemometry and Thermocouple Pyrometry," NACA TN 3514, September, 1955.

3 Thermo-Systems, Inc., Brochure on the Heat Flux System.

TABLE I
DATA FOR FIGURES

CIRCUIT

G_V = 1,000

R/R_L = 1 (Figure 4)

E = 4 Volts (Figure 5)

High Sensitivity Bridge

R_3 = 40 ohms

G_I = 0.95

High Power Bridge

R_3 = 2 ohms

G_I = 0.90

ENVIRONMENT
(Figures 4 and 7)

Velocity = 200 ft/sec

Temperature = 70°F

Heat Transfer Coefficients

d_o	h_o (BTU/hr-ft^2-°F)
0.006 in.	600
0.001 in.	1480
0.00 in.	3280

SENSOR

Properties

Glass

ρ_g = 159 lbs/ft^3

c_{p_g} = 0.195 BTU/lb-°F

k_g = 1.17 BTU/hr-ft-°F

Platinum Wire

ρ_w = 1334 lbs/ft^3

c_{p_w} = 0.0365 BTU/lb-°F

Lengths

d_o:0.006 in., = 0.10 in.

d_o:0.001 in., = 0.03 in.

d_o:0.0002 in., = 0.08 in.

Resistance

R_p = 20 ohms

Figure 4

R_r = 7.12 ohms, = 0.00166/°F (t_s=800°F)

Figure 5

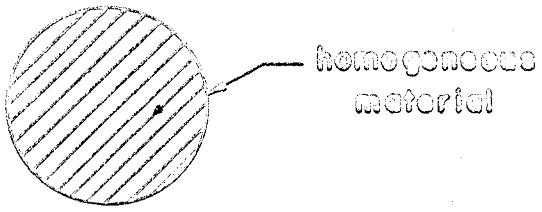
R_r = 11.45 ohms, = 0.00166/°F (t_s=450°F)

Figure 7

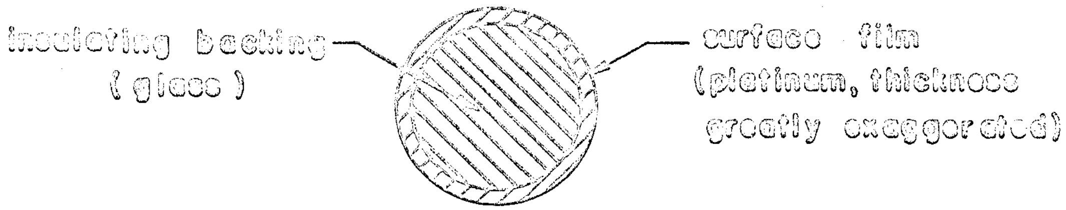
R_r = 13.45 ohms, = 0.0007/°F (t_s=800°F)

Figure 8

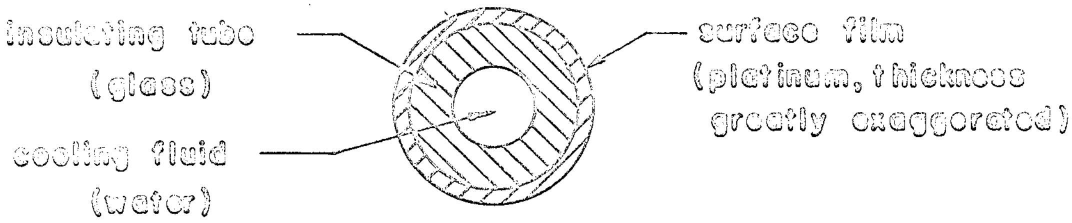
R_r = 16 ohms, = 007°F (t_s=450°F)



HOT WIRE ELEMENT

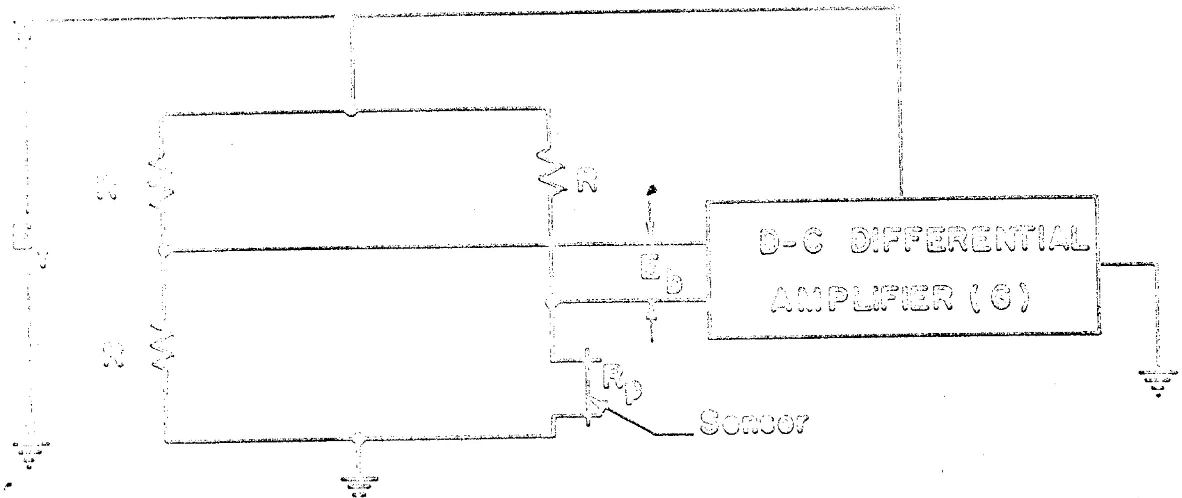


HOT FILM ELEMENT

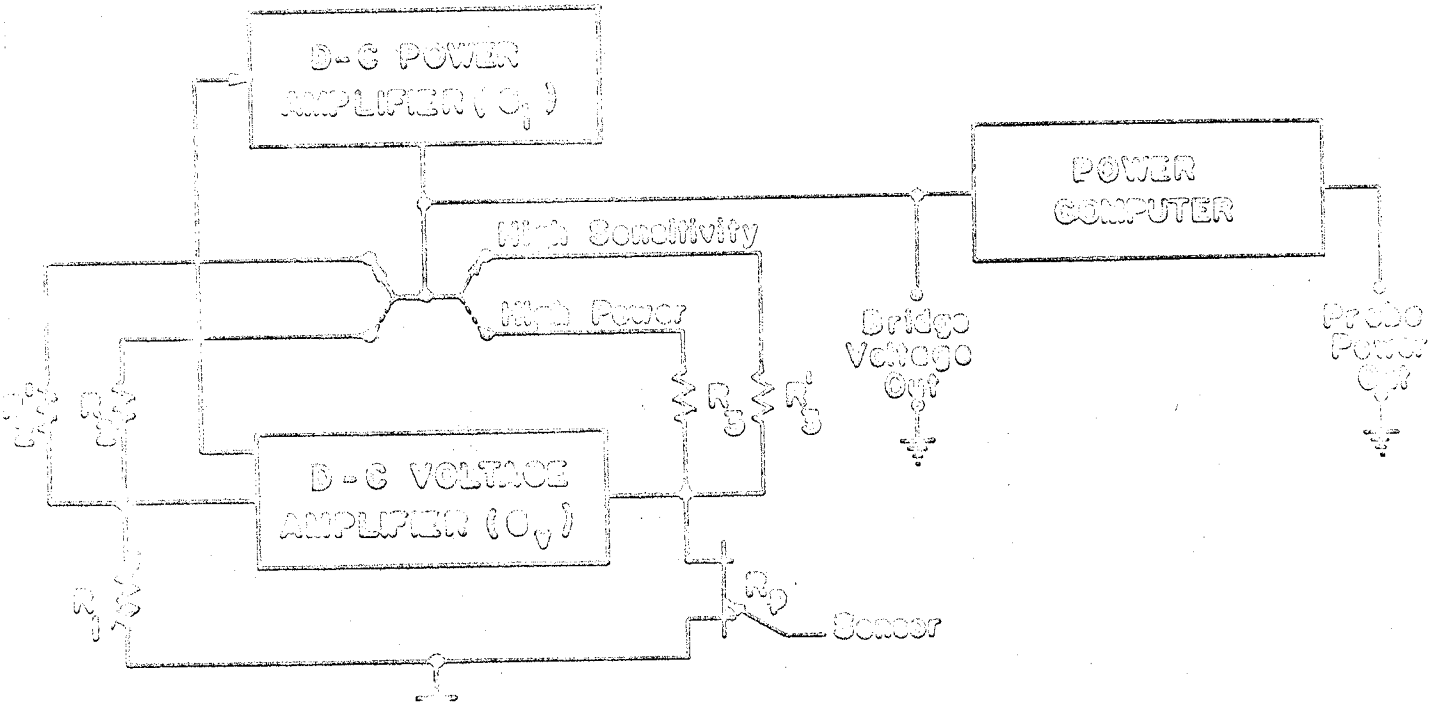


COOLED FILM ELEMENT

FIGURE 1. CROSS-SECTIONS OF SENSORS



(a) STANDARD CONSTANT TEMPERATURE COMPENSATING CIRCUIT



(b) HEAT FLUX SYSTEM WITH FLOATING AMPLIFIER

FIGURE 2. CONSTANT TEMPERATURE SYSTEMS

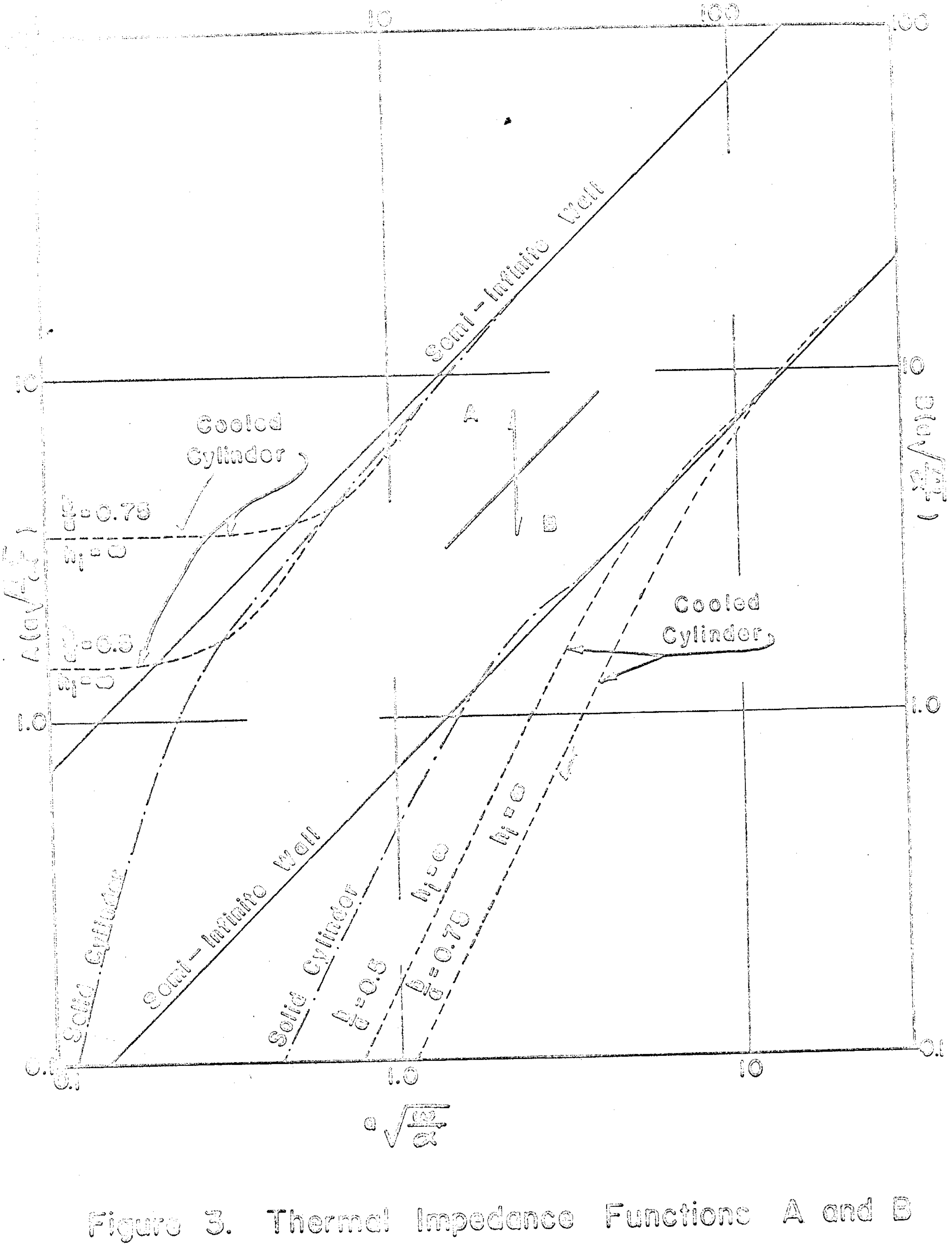


Figure 3. Thermal Impedance Functions A and B

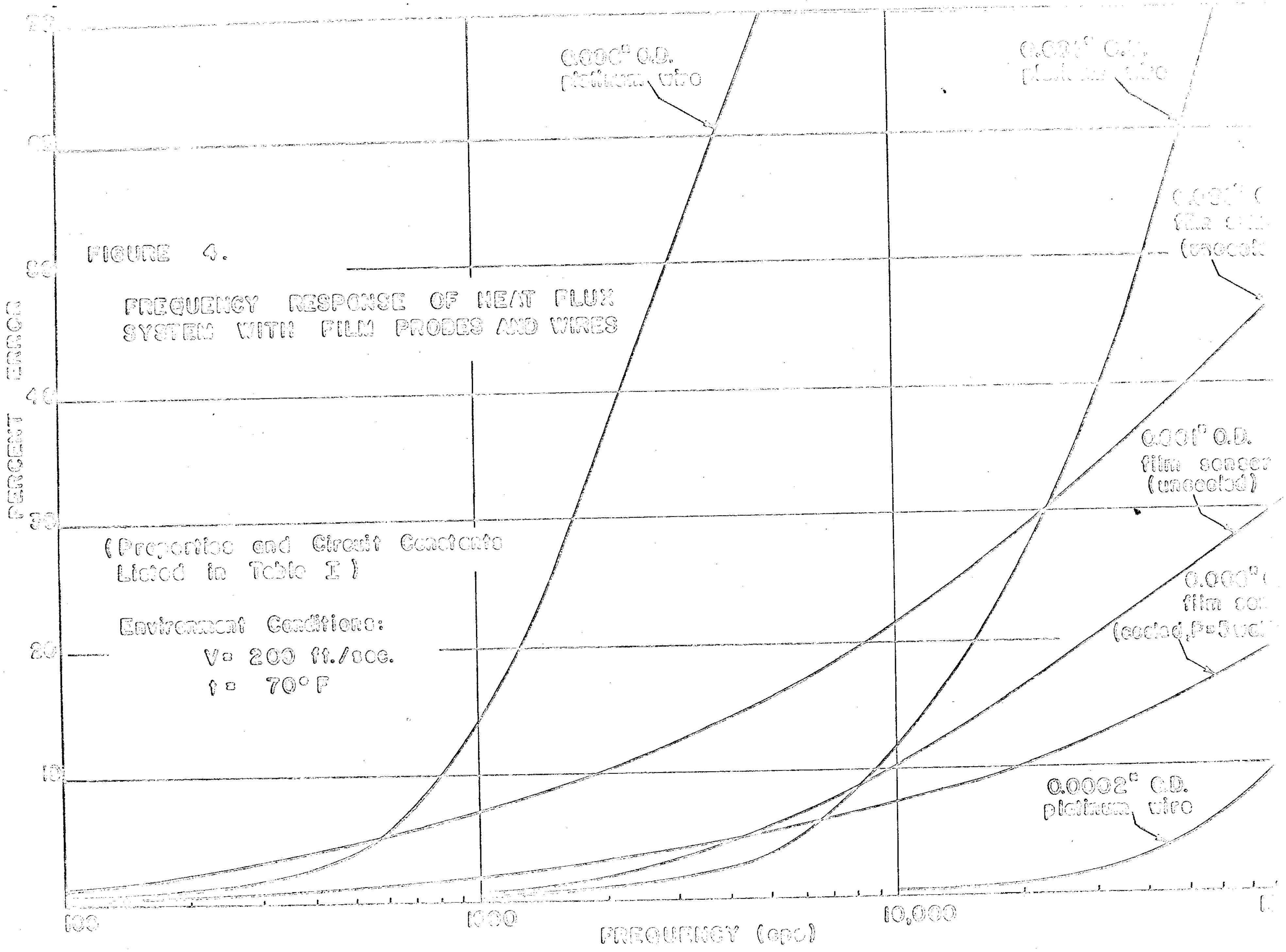


FIGURE 4.

FREQUENCY RESPONSE OF HEAT FLUX SYSTEM WITH FILM PROBES AND WIRES

(Properties and Circuit Constants Listed in Table I)

Environment Conditions:
V= 200 ft./sec.
t= 70° F

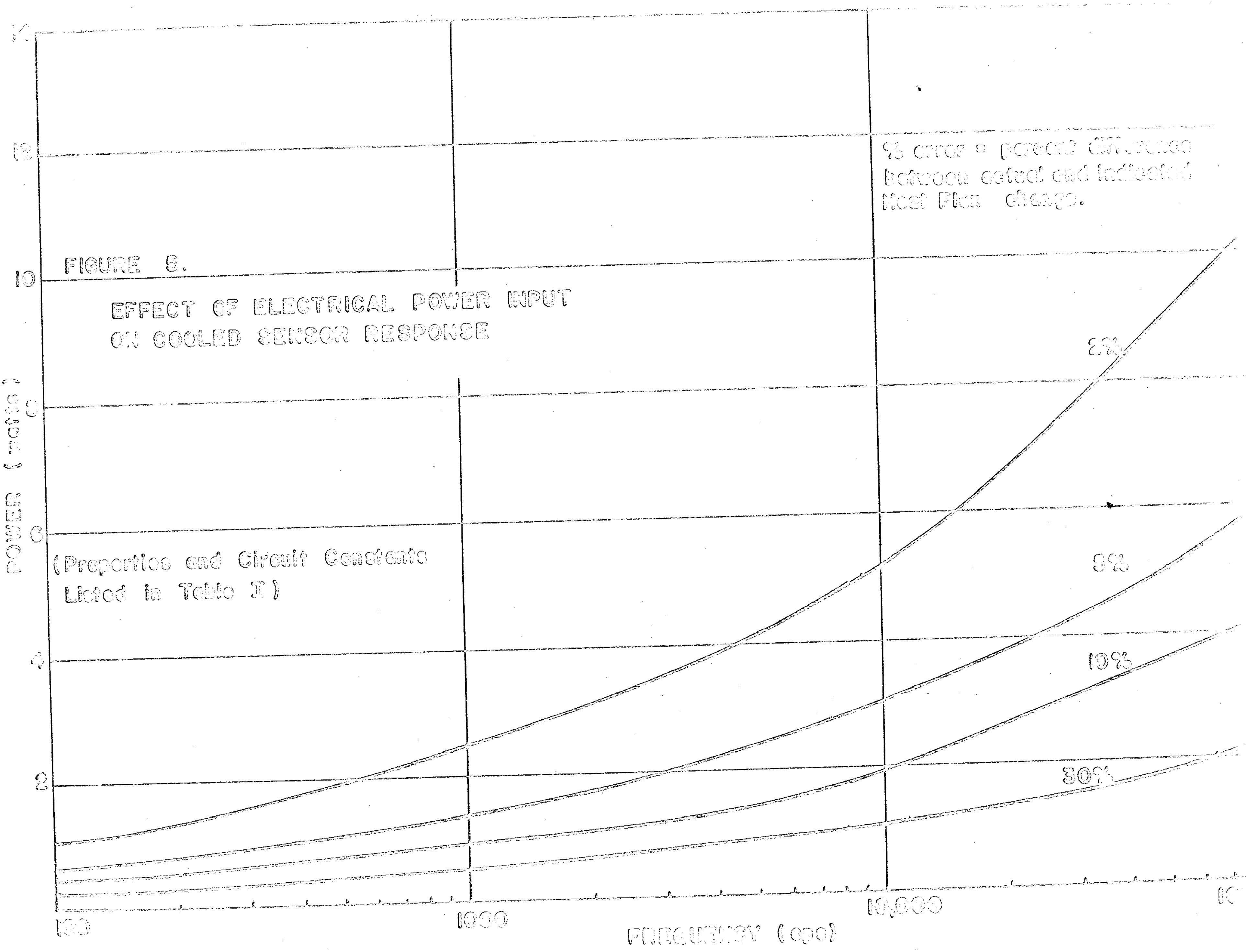


FIGURE 5.

EFFECT OF ELECTRICAL POWER INPUT
ON COOLED SENSOR RESPONSE

(Properties and Circuit Constants
Listed in Table I)

% error = percent difference
between actual and indicated
Heat Flux change.

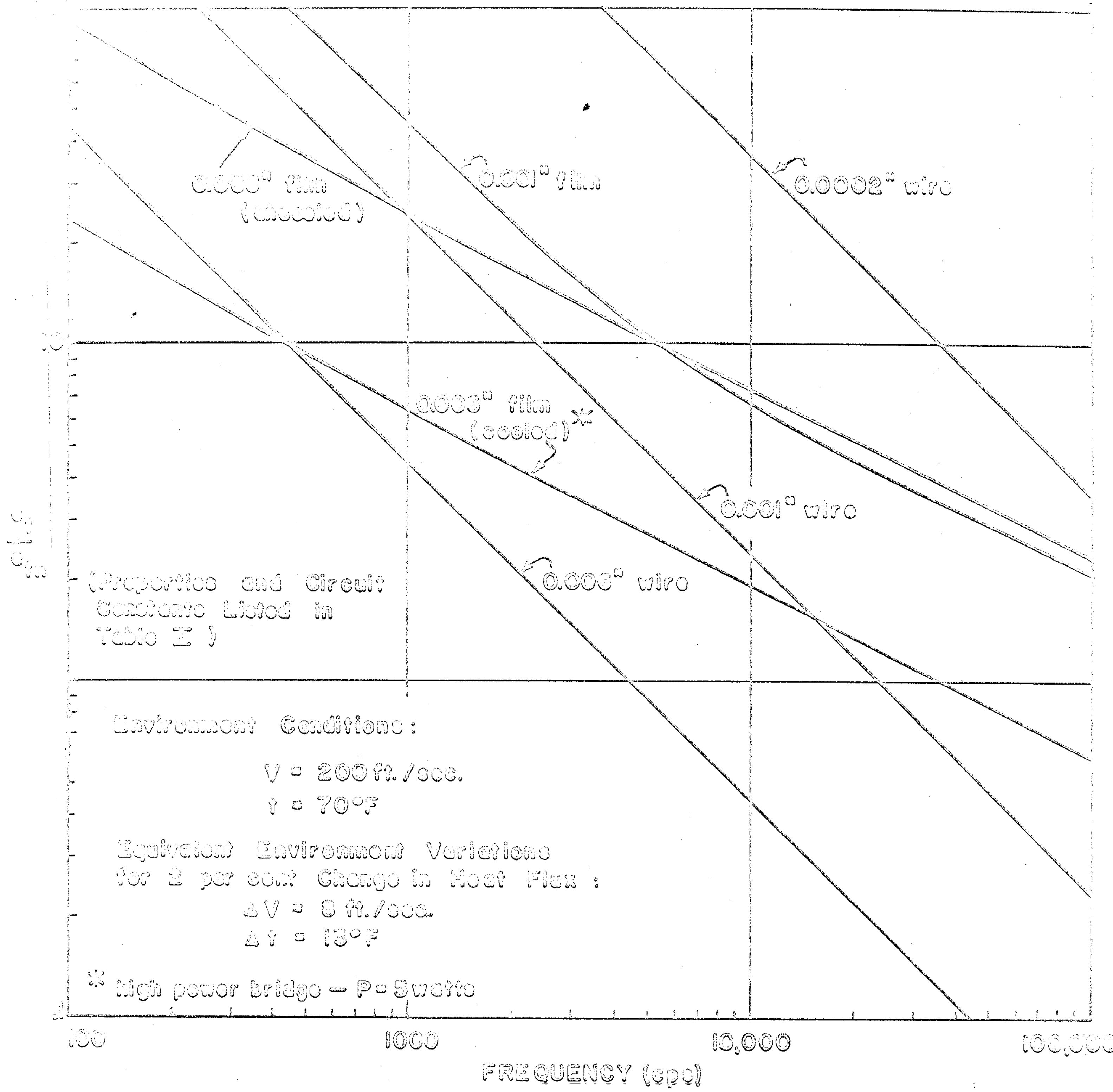
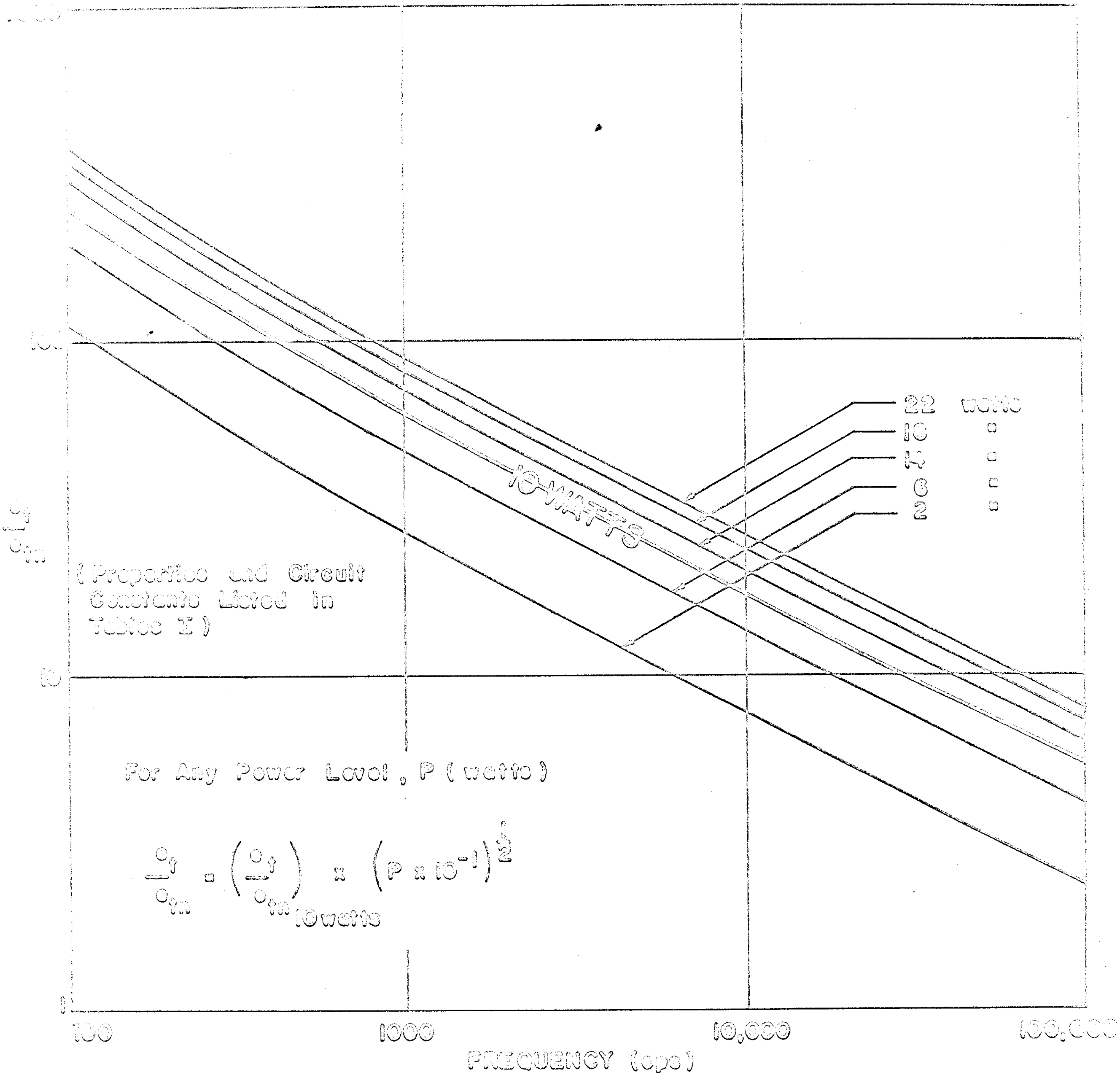


FIGURE 6. RELATIVE SIGNAL TO NOISE RATIO FOR HEAT FLUX VARIATION OF 2 PERCENT OF MEAN VALUE (e_{bn} = 20x10^{-6} volts)



(Properties and Circuit Constants Listed in Tables I)

For Any Power Level, P (watts)

$$\frac{e_t}{e_{tn}} = \left(\frac{e_t}{e_{tn}}\right)_{10\,\text{watts}} \times \left(P \times 10^{-1}\right)^{\frac{1}{2}}$$

FIGURE 7. RELATIVE SIGNAL TO NOISE RATIO FOR 0.10 WATT VARIATION IN HEAT FLUX TO SENSOR ($e_{bn} = 20 \times 10^{-6}$ volts)