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STRIATED COMBUSTION SOLUTION

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FOREWORD

This report presents the results of work performed by Lockheed's Huntsville Research & Engineering Center while under subcontract to Northrop Nortronics (NSL PO 5-09287) for the Aero-Astrodynamics Laboratory of Marshall Space Flight Center (MSFC), Contract NAS8-20082. This task was conducted in response to the requirement of Appendix B-1, Schedule Order No. 104, "Nozzle and Jet Wake Study."

SUMMARY

Consideration of full striation effects is a necessary part of an accurate flowfield analysis for liquid rocket engines. One of the tasks currently in use at Lockheed/Huntsville is a finite-rate/mixing program designed for combustor analysis. An equilibrium solution capable of handling stream tubes operating at different oxidizer/fuel ratios is necessary to provide realistic starting conditions for this finite-rate analysis. The program may also be used to give meaningful information concerning the consequences of introducing fuel striations from the performance standpoint.

A parametric study of the amount of fuel used for film cooling in a liquid oxygen/RP-1 combustion system is presented. The predictions are compared with F-1 engine data.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

Symbol	Definition
A	area
A_c	combustion chamber area
A_s	stream area
a	defined in text
C_p	normalized specific heat at constant pressure
C_1, C_2, C_3	constants defined in text
ds	differential step length
$d\bar{s}$	differential step vector
e	error vector
$\bar{e}_s$	step direction vectors
f	function to be minimized
$\dot{m}$	mass flow rate
p	pressure
p^{**}	pressure after combustion
p_I	injector pressure
R	gas constant
T	temperature
T^{**}	temperature after combustion
u	velocity

Nomenclature (Continued)

Symbol	Definition
Greek	
ρ	density
Superscripts	
$'$	condition at injector face
$\sim$	denotes quantity in physical units
j	iteration number
$*$	reference conditions
Subscripts	
α	pertaining to the α^{th} stream
β	pertaining to independent variable
Others	
$\vert \rangle$	column vector
$\langle \vert$	row vector
$\langle \vert \rangle$	inner product

Section 1
INTRODUCTION

In the study of rocket engines it is sometimes necessary to consider spatial propellant variations. These variations are purposely induced in the case of film cooling of chamber walls or are inadvertently introduced in the injector design. The fuel striation phenomenon may have a pronounced effect on the performance and exhaust characteristics of the jet discharge.

A sophisticated finite rate-mixing program exists (Reference 1) which may be used to predict the reactions and mixing occurring in a combustor which contains fuel striations. Unfortunately, this computer program requires an ignition source of such a magnitude that the predictions are compromised. An alternate method is to perform an equilibrium combustion calculation and allow the subsequent mixing controlled reactions to progress at finite rate. A requirement thus exists for an equilibrium combustion analysis.

This report contains a discussion of the analytical techniques employed in a computer program which was created to perform the multiple stream tube calculation. A parametric study of the F-1 engine is discussed to illustrate the uses of the program.

Section 2
TECHNICAL DISCUSSION

In order to provide initial conditions for use in the finite rate reaction/mixing program discussed in Reference 1, an equilibrium combustion analysis is necessary. A discussion of the method used to solve the problem and the resultant computer program follows.

Consider a multistream injection into a cylindrical combustor as shown in Figure 1. The entering propellants ignite and expand, rapidly accelerating to the end of the combustion region. If there are radial mixture variations, combustion properties will vary and each stream tube will attain different speeds. This results in a shearing action between the stream tubes and is the principal cause of the subsequent mixing process.

In order to determine the conditions just downstream of the combustion region, the following assumptions are made:

- the reaction rates are fast enough that the gases can be considered in chemical equilibrium
- no radial pressure variations exist
- mixing effects may be ignored upstream of the combustion front
- each stream behaves ideally away from the adiabatic flame condition for that mixture
- all inlet conditions are known.

The third assumption is a consequence of the first assumption and the realization that the initial combustion region is probably very short compared to the overall combustor length.

Development of Governing Equations

Under the above assumptions the governing equations between the injector face and the combustion termination station may be written.

Conservation of mass:

$$(\rho u A)_\alpha - \dot{m}_\alpha = 0 \qquad ; \qquad \alpha = 1, n \tag{1}$$

Conservation of momentum:

$$(\bar{p} A)'_\alpha - p A_\alpha - C_1 \dot{m}_\alpha u_\alpha = 0 \qquad ; \qquad \alpha = 1, n \tag{2}$$

Conservation of energy:

$$T_\alpha = T_{0\alpha} - C_{2\alpha} u_\alpha^2 ; \quad \alpha = 1, n \tag{3}$$

Also, we may write the equation of state:

$$\rho_\alpha = C_{3\alpha} p / T_\alpha \tag{4}$$

and a geometric relation:

$$\sum_{\alpha=1}^{n} (A_\alpha - A'_\alpha) = 0 \tag{5}$$

For improved numerical behavior the above equations have been normalized in the following manner

$$\rho = \widetilde{\rho}/\rho_1^*; \; u = \widetilde{u}/u_1^*; \; p = \widetilde{p}/p_I; \; A = \widetilde{A}/A_c$$

so that

$$\dot{m} = \tilde{\dot{m}}/\rho_1^* u_1^* A_c$$

while

$$C_1 = \rho_1^* u_1^{*2}/p_I$$

$$C_{2\alpha} = u_1^{*2}/2\, C_{p_\alpha}$$

$$C_{3\alpha} = p_I/R_\alpha\, \rho_1^*$$

Now Equations (1), (3) and (4) may be readily combined to yield

$$p\left[C_3\, u\, A/(T_o - C_2\, u^2)\right]_\alpha - \dot{m}_\alpha = 0 \qquad ; \qquad \alpha = 1, n \quad (6)$$

Equations (2), (5) and (6) represent 2 n+1 equations in a like number of unknowns. A suitable mechanism for solving this set of equations must now be found.

Let the row vector $< x \,|$ be

$$< x \,| = (A_1, \ldots, A_n, u_1, \ldots u_n, p)$$

For any arbitrary guess of $< x \,|$ each of the governing equations will be somewhat in error. The error vector will be denoted by $|\, e >$ and a function f is defined as

$$f = <e|e> \qquad (7)$$

Now f will be positive definite and will achieve a minimum value (zero) at the desired solution point. In order to systematically proceed from an arbitrary initial estimate to the desired solution we may employ the steepest descent. In this free minimization problem we may write

$$df = \nabla f \cdot d\bar{s}$$

where $d\bar{s}$ is a small step away from the present position. In order to maximize the payoff or change in the function for a given magnitude $|d\bar{s}|$ we must maximize the dot product below.

$$df = \frac{(\nabla f \cdot \bar{e}_s)|\nabla f| ds}{|\nabla f|}$$

hence

$$\bar{e}_s = \frac{\nabla f}{|\nabla f|}$$

Now the desired change in the function is

$$df = 0 - f = -f$$

so that

$$| x >^{j+1} = | x >^{j} - \frac{f^j}{|\nabla f^j|^2} \left| \frac{\partial f^j}{\partial x\beta} \right> \qquad (8)$$

Now $\left| \frac{\partial f}{\partial x\beta} \right>$ is found by

$$\left| \frac{\partial f}{\partial x\beta} \right> = 2 \left| < e \left| \frac{\partial e}{\partial x\beta} \right> \right>$$

The error vector $| e >$ is formed by

$$e_1 = \rho_1 u_1 A_1 - \dot{m}_1$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$e_n = \rho_n u_n A_n - \dot{m}_n$$

$$e_{n+1} = (\bar{p} A)'_1 - p A_1 - \dot{m}_1 u_1$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$e_{2n} = (\bar{p} A)'_n - p A_n - \dot{m}_n u_n$$

$$e_{2n+1} = \sum_{\alpha=1}^{n} (A_\alpha - A'_\alpha)$$

The vector $\langle \frac{\partial e}{\partial x \beta} |$ is

$$\langle \frac{\partial e}{\partial x \beta} | = \langle a_1, \ldots, a_\alpha, \ldots a_n, a_{n+1}, \ldots a_{n+\alpha}, \ldots a_{2n}, a_{2n+1} |$$

where, for $1 \leq \beta \leq n$

$$a_\alpha = \begin{cases} \rho_\alpha u_\alpha & \beta = \alpha \\ 0 & \beta \neq \alpha \end{cases} ; \quad a_{n+\alpha} = \begin{cases} -p & \beta = \alpha \\ 0 & \beta \neq \alpha \end{cases} ; \quad a_{2n+1} = 1$$

and for $n + 1 \leq \beta \leq 2n$

$$a_\alpha = \begin{cases} \rho_\alpha A_\alpha + u_\alpha A_\alpha \dfrac{\partial \rho_\alpha}{\partial u_\alpha} & \beta - n = \alpha \\ 0 & \beta - n = \alpha \end{cases} ; \quad a_{n+\alpha} = \begin{cases} -\dot{m}_\alpha & \beta - n = \alpha \\ 0 & \beta - n \neq \alpha \end{cases} ; \quad a_{2n+1} = 0$$

while for $\beta = 2n+1$

$$a_\alpha = \left(\frac{u\,A}{R\,T}\right)_\alpha ; \quad a_{n+\alpha} = -A_\alpha ; \quad a_{2n+1} = 0$$

where

$$\frac{\partial \rho_\alpha}{\partial u_\alpha} = \left[2\, C_2\, \rho\, u / (T_o - C_2\, u^2) \right]_\alpha$$

The equations above have been programmed in Fortran IV language. As additional output, the choking condition and supersonic expansion calculations have also been provided. The expansion after combustion is considered isentropic and inviscid. The choking condition is defined as the minimum area obtainable (satisfying energy and conservation). The static pressure after combustion is p^{**} while the temperature is T_α^{**} for each stream tube. Then rewriting (1) yields

$$A_\alpha = \left(\frac{\dot{m}}{\rho\, u}\right)_\alpha$$

and the stream area is

$$A_s = \sum_{\alpha=1}^{n} A_\alpha$$

By utilizing the equations previously presented and the isentropic expansion relation

$$T_\alpha = T_\alpha^{**} \left(\frac{p}{p^{**}}\right)^{\frac{\gamma_\alpha - 1}{\gamma_\alpha}}$$

the minimum value of A_s may be found. After the choking condition had been found the pressure is continually decreased until a specified value of exit area ratio is reached.

Discussion of Sample Case

To illustrate the effects of fuel striations as idealized by this analysis, a sample calculation representative of the F-1 engine is discussed. The necessary operating conditions are

p_I	$.15 \times 10^6$	psf
$\dot{m}$	$.15 \times 10^3$	slugs/sec
O/F	2.3	
A_c	8.3	ft^2
propellant	LOX/RP-1	

The injection momentum was assumed to be zero. A two-stream solution in which a wall stream of raw fuel and a main stream operating at various O/F ratios was performed. The total stream O/F ratio was maintained at 2.3, however. In this analysis the injector area devoted to each stream was assumed to be proportional to the mass flow of that stream. The thermodynamic properties of each stream were taken from the NASA/Lewis Thermochemical Program, Reference 2. In the case of the wall stream (fuel stream) the inlet temperature was chosen as 421^oK. This value should be typical of the inlet temperature since a portion of the fuel is heated during the regenerative cooling cycle.

Figure 2 illustrates that a growth of the wall or fuel stream tube occurs during combustion. Since this is a constant area duct, the main stream shrinks to accommodate this expansion. The assumed injection area and final combustion area are plotted as a function of the main stream O/F ratio.

Since the two streams have different properties, it is only natural that the choking condition will occur at slightly different than sonic speeds for both streams. Because the main stream is large compared to the fuel stream, the minimum throat and the sonic condition occur almost simultaneously. Figure 3, however, shows that in a typical case, the minimum area occurs while the wall stream is still subsonic. Choking of the wall stream occurs downstream of the physical throat. Although the Mach numbers of each stream are reasonably close, there is a large velocity discrepancy due to the large difference in total temperatures of the streams.

Figure 4 indicates that the choking area is strongly affected by the variation of main stream O/F ratio. The actual F-1 minimum area occurs at a main stream O/F ratio of 2.65.

Figure 5 describes the vacuum impulse predicted by the program as a function of exit area ratio for various main stream O/F values. Figure 6 shows the impulse calculated at the actual F-1 exit area. The F-1 nominal impulse occurs at a main stream O/F of about 3.05.

The nominal F-1 values indicate that the engine operates at a main stream O/F that is higher than the overall. The discrepancy (2.65 by choking relationship and 3.05 by impulse comparison) between the two values undoubtedly lies in the oversimplification of the combustion/expansion model. These calculations are more appropriately viewed as indicative of trends rather than precise predictions.

Section 3
CONCLUSIONS

An equilibrium multiple stream tube combustion solution has been developed and programmed. Although its primary function is to provide input to Reference 1, the sample calculation illustrates its usefulness in an idealized performance calculation. Minor improvements in the stream tube model could be made, but it is felt that they would be unwarranted.

It would appear, however, that the impact on performance is great enough that a major development would be warranted. A two-dimensional viscous, equilibrium solution from the injector face all the way to some moderately supersonic station is technically feasible and would vastly improve the state-of-the-art performance and exhaust descriptions.

REFERENCES

1. Edelman, R. and O. Fortune, "Mixing and Combustion in the Exhaust Plumes of Rocket Engines Burning RP-1 and Liquid Oxygen," Technical Report No. 631, General Applied Sciences Laboratories, Inc., November 1966.

2. Zeleznik, F. J. and S. Gordon, "A General IBM 709 and 7090 Computer Program for Computation of Chemical Equilibrium Compositions, Rocket Performance, and Chapman-Jouguet Detomations," NASA TN D-1454, October 1962.

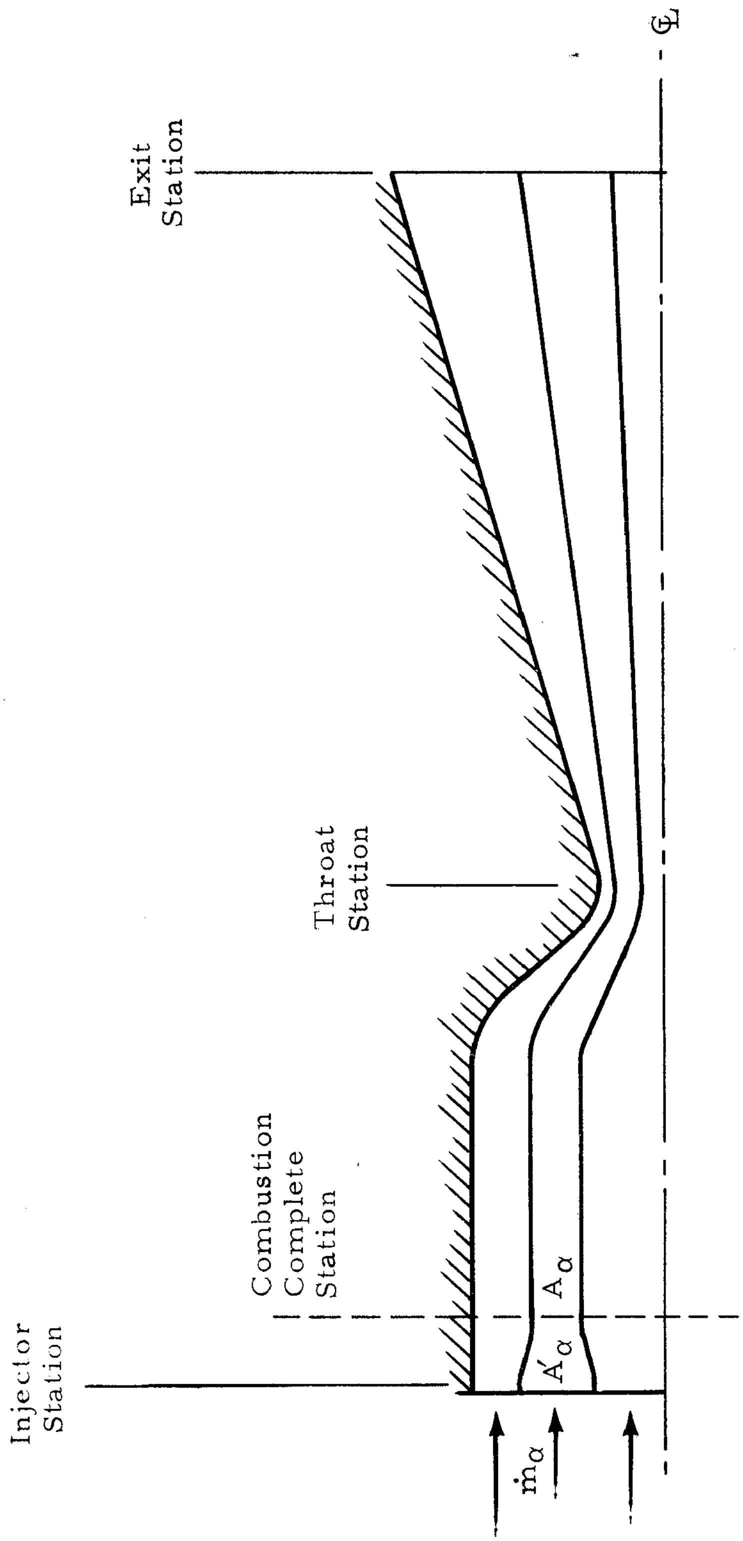


Figure 1 - Rocket Engine Schematic

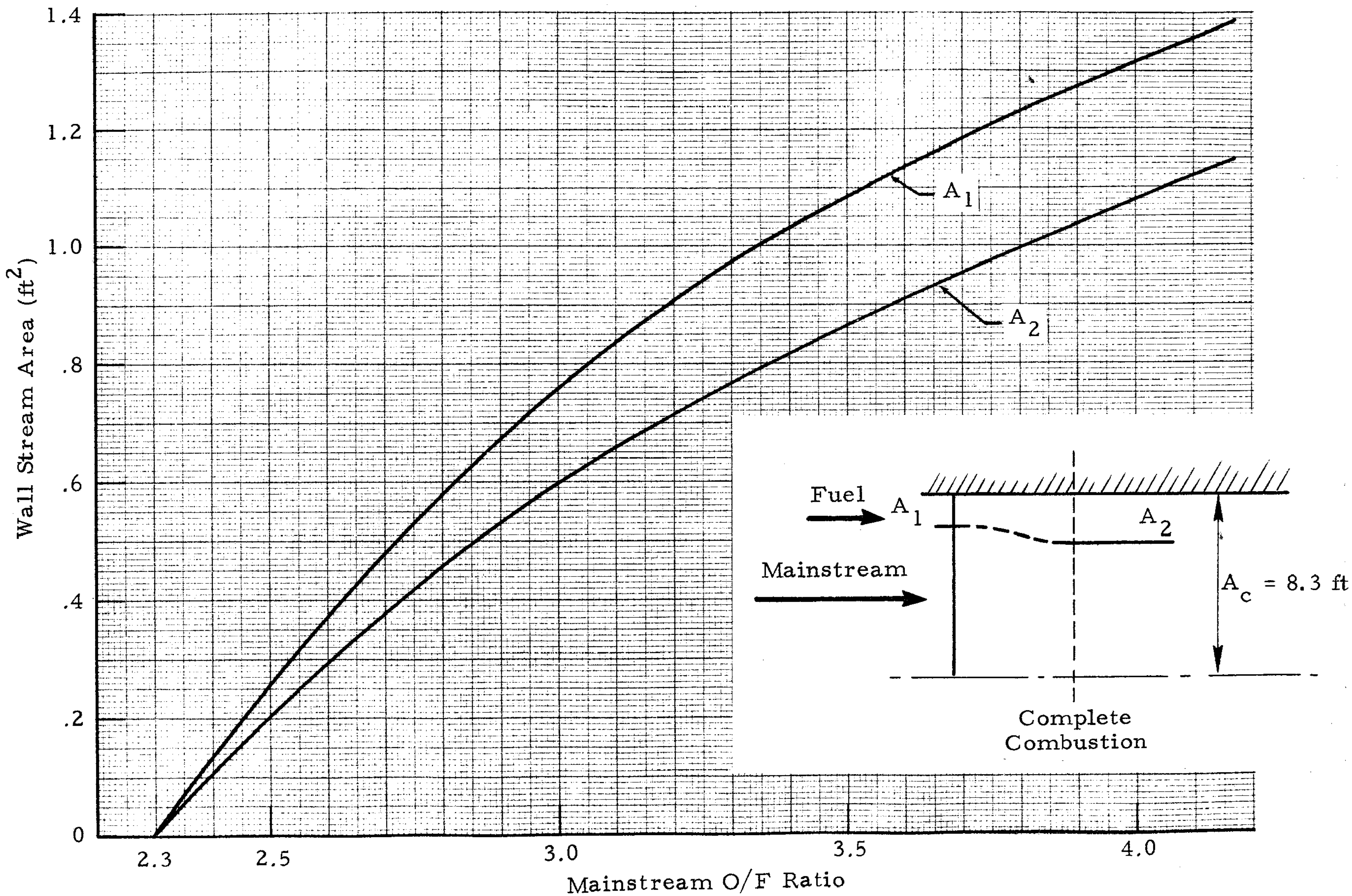


Figure 2 - Change in Wall (Fuel) Stream Area During Combustion in F-1 Combustion Chamber

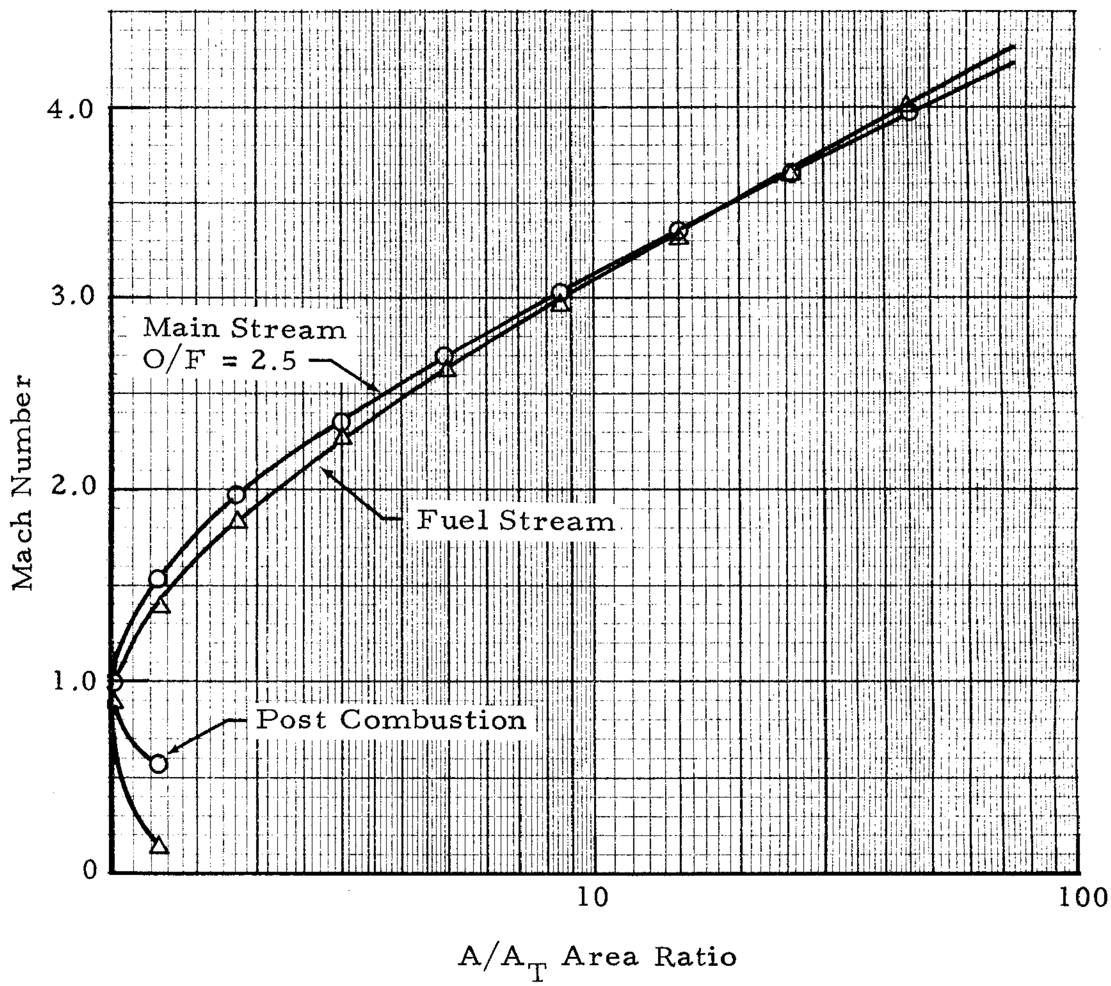
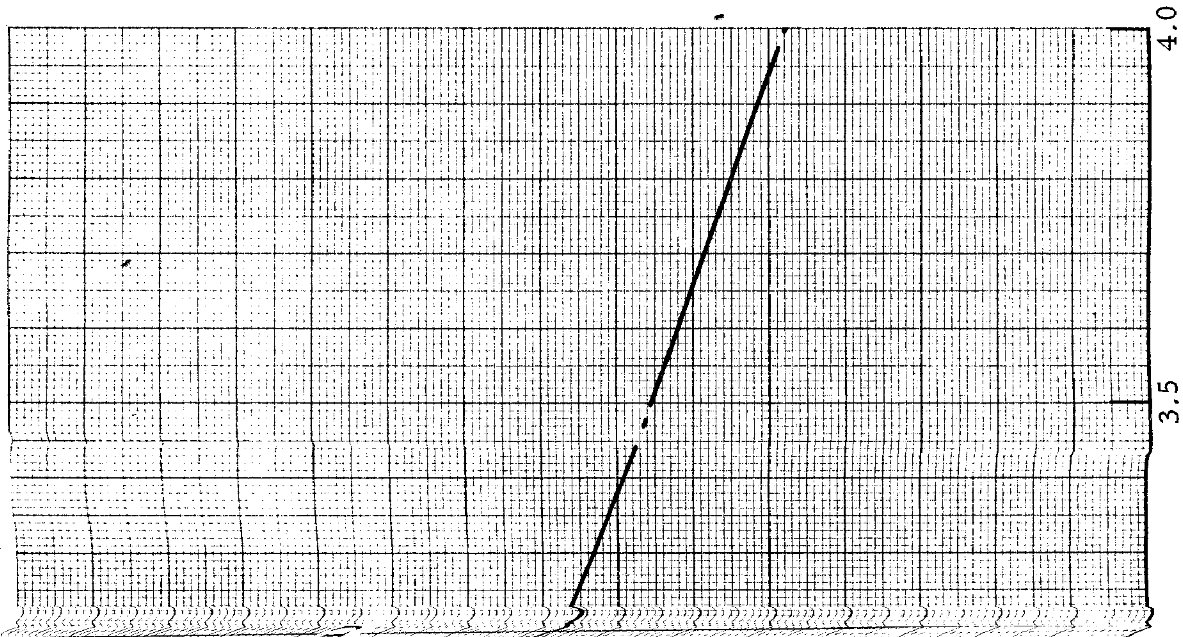


Figure 3 - Variation of Stream Mach Number as a Function of Area Ratio for Mainstream O/F = 2.5



...ea with Mainstream O/F

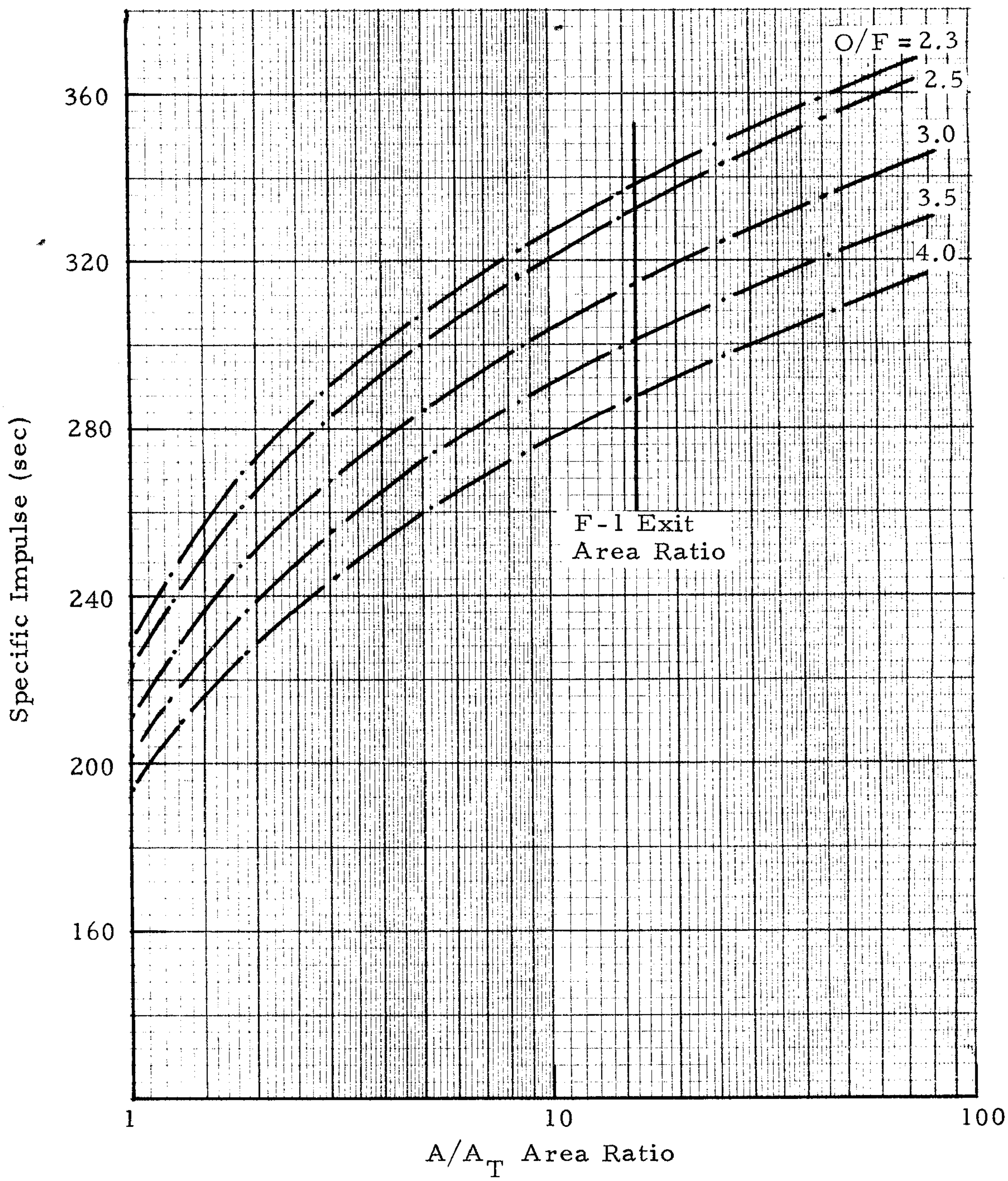


Figure 5 - Variation of Specific Impulse with Nozzle Exit Area Ratio for Several Mainstream O/F Values

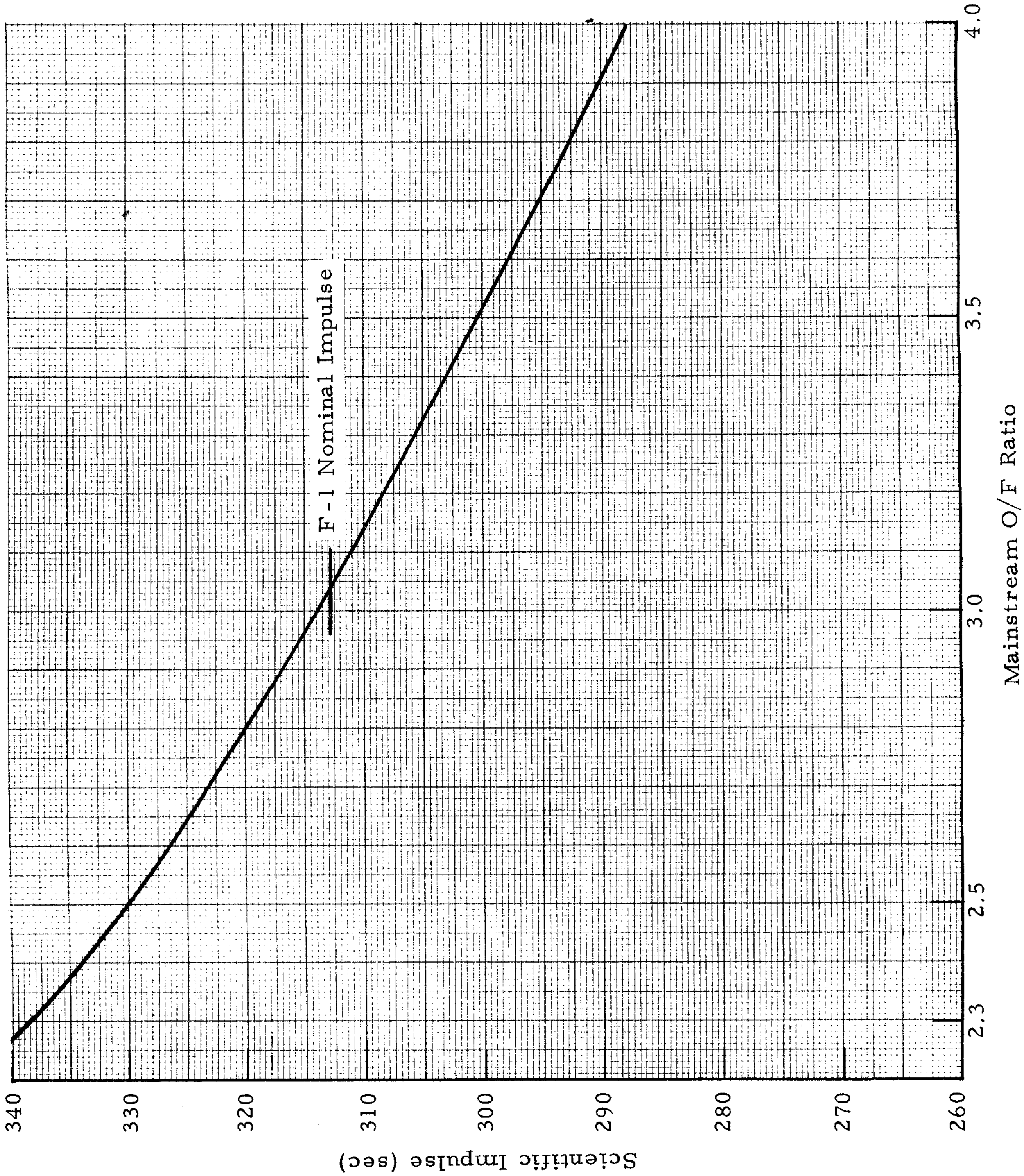


Figure 6 - Variation in F-1 Impulse with Mainstream O/F Ratio