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Flame Contact Studies

II. Experimental Validation of Mathematical
Analysis of Heat Flow

Bureau of Medicine and Surgery
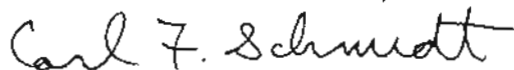Subtask MR005.13-1005.1   Report No. 29

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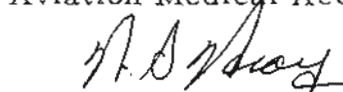
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SUMMARY

Mathematical equations for transient heat flow through a two-layer wall are examined, corrected, and validated experimentally for application to thin layers of material (0 to 1 mm thick) during short-term heating (3 sec) in gas flames.

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INTRODUCTION

An apparatus and a method for the study of heat transfer through fabrics in contact with gas flames at 1200°C have been described (1). Data obtained directly from experimental observations were given for destruction temperature, resistance to heat transfer, and the insulation effect of air spaces between layers of fabrics woven of a new high-temperature-resistant fiber (DuPont HT-1). The present report concerns the establishment of a system for the mathematical analysis of these experimental data whereby surface and interface temperatures, temperatures in depth, and various properties of the fabrics may be derived. The equations examined are those for transient heat flow through a two-layer wall as proposed by Griffith and Horton (2).

PROCEDURE AND MATERIALS

Due to the total dependence to be placed upon the mathematical expressions chosen to represent the heat flow situation, it was necessary to validate the analytical process experimentally. The validation required a substance of known thermal properties that could be obtained in fairly thin sheets, would not wrinkle or swell appreciably on flame contact, and would form a tight contact with the backing material. Silicone rubber* was selected for this purpose. Bubble-free sheets of this material were molded

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* RTV-20 manufactured by the General Electric Company

on a plastic surface in thicknesses from about 0.5 to 1.0 mm. The plastic imparted a smooth surface to the rubber which, due to its surface properties, formed an intimate contact with the backing material without the use of a filming or filling agent. (The backing material was the simulated skin device made by the Naval Material Laboratory, N.Y., and previously described[3].) .

Exposures of the backed silicone rubber were made until the temperature rise at a fixed time during exposure, at a fixed depth beneath the surface of the simulated skin, was established for several thicknesses of the rubber. In the equations of Griffith and Horton the expression for this temperature rise was given as (Equation 1, Appendix):

$$U_2 = \frac{2H\lambda\sqrt{D_1}}{\gamma} \sum_{n=0}^{n=\infty} \left(-\frac{1}{\gamma}\right)^n \left\{ 2\sqrt{\frac{D_2 t}{\pi}}\; e^{-\left\{x-\alpha\left\{1-\sqrt{D_2/D_1}\,(2n+1)\right\}\right\}^2/4D_2 t} \right.$$

$$\left. -\left[x-\alpha\left\{1-\sqrt{D_2/D_1}(2n+1)\right\}\right]\left(1-\mathrm{erf}\frac{x-\alpha\left\{1-\sqrt{D_2/D_1}(2n+1)\right\}}{2\;D_2 t}\right)\right\}$$

where

subscript 1 refers to the outer layer (silicone rubber)

subscript 2 refers to the base layer (simulated skin)

and

U = temperature rise

H = heat flux perpendicular to surface

x = total thickness from surface to point of temperature rise measurement

α = thickness of layer 1

D = thermal diffusivity = k/S

k = thermal conductivity

S = volume specific heat (density x specific heat)

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$$\gamma^* = \frac{k_2 S_2 + \sqrt{k_1 S_1 k_2 S_2}}{k_2 S_2 - \sqrt{k_1 S_1 k_2 S_2}}$$

$$\lambda = (k_2 \sqrt{D_1} - k_1 \sqrt{D_2})^{-1}$$

$$t = \text{time}$$

*As corrected

As seen from this equation, in addition to the thermal constants for both layers, the heat flux and the thickness of the layers must be known in order to compare the computed value with the experimental value. The heat flux was measured directly by means of a flame impingement calorimeter[**] (4) in the classical manner of water calorimetry as described in detail elsewhere (5). This instrument had been designed to measure much higher fluxes so that very low rates of flow had to be maintained in order to produce an appreciable temperature difference between the incoming and outgoing water. Elimination of bubbles in the water system was difficult but was aided by addition of a small amount of wetting agent (5 drops of a 10% solution of benzalkonium chloride per liter of reservoir water). Routinely, the calorimeter was placed with the sensing element at the level of the experimental specimen and heated until equilibrium was reached as indicated by a constant temperature difference between the inflow and outflow water. The outflow was collected over a carefully clocked

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period of about 10 seconds and weighed. The rate of flow was then determined and the heat flux calculated from the relationship:

$$H = \frac{{}^{o}C \times cc/sec}{1.268}$$

where H = heat flux $(cal/cm^2 sec)$

^{o}C = temperature difference between inflow and outflow

cc/sec = rate of flow

1.268 = area of sensing element (cm^2)

The heat flux was determined, also, indirectly by the temperature rise at depth in a substance of known thermal properties, the simulated skin. In this procedure the bare simulated skin was exposed to the flame for 3 seconds and the temperature rise at depth noted. Then the heat flux was determined from the relationship between temperature rise at a known depth and heat flux perpendicular to the surface (5).

$$H = \frac{\Delta To}{36.2}$$

where H = heat flux in $cal/cm^2 sec$

ΔTo = temperature rise at depth 0.049 cm at 3 sec in ^{o}C

36.2 = temperature rise in ^{o}C/unit heat flux given for the simulated skin (3) and also calculable by Eq. 1 when $\alpha = 0$.

The thickness of the silicone rubber slabs was determined from sections carefully cut from the area which had been located above the embedded thermo-couple during the heat exposures. Some measurements were made with a

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dissecting microscope on unmounted sections and others were made at

higher magnifications on paraffin-mounted sections.[**]

For short exposure times, the simulated skin provided an essentially semi-infinite base layer. The embedded thermocouple was 0.001 cm thick and was situated at a level 0.050 cm beneath the surface of the simulated skin; thus, for purposes of calculation, the thickness of the second layer was 0.049 cm.

Of the material constants required for application of the equations, only the density was corroborated by measurements made in this laboratory; the diffusivity, volume specific heat, and thermal conductivity values used were those supplied by the manufacturers or computed from the data supplied.

EXPERIMENTAL DATA AND DISCUSSION

The pertinent physical and thermal properties of the silicone rubber and of the simulated skin are shown in Table I.

TABLE I

Property	Units	Silicone Rubber	Simulated Skin
Thickness	cm	0.050 – 0.100	1.0
Density	gm/cm^3	1.35	1.82
Vol. specific heat	$cal/cm^3 {}^oC$	0.47	0.65
Diffusivity	cm^2/sec	13.5×10^{-4}	20.3×10^{-4}
Thermal conductivity	$cal/cm\ {}^oC\ sec$	6.4×10^{-4}	$\frac{13.1}{31.1} \times 10^{-4}$

Measurements of the heat flux by means of the flame impingement calorimeter proved to be quite variable as might be expected since the instrument had been designed to measure flow levels fifty times as high as those concerned here. The calorimeter water flow ranged from 0.20 to 0.47 cc/sec and the temperature difference, from 3.46 to 10.42^oC. The average of thirty apparently reliable measurements was 1.61 cal/cm^2 sec ± 0.25, gross deviation with a standard deviation (σ) of ± 0.12^oC. Since all systematic errors would be in the direction of lowering this value, the ten highest measurements were averaged separately. These yielded a value of 1.74 cal/cm^2 sec ± 0.12, gross deviation.

Ten measurements of heat flux made with the simulated skin yielded an average value of 1.73 cal/cm^2 sec ± 0.02, gross deviation. This method proved to be far more convenient, sensitive and accurate than

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the calorimetric procedure at this level of flux. Therefore, the value of

1.73 cal/cm^2 sec was accepted as correct while the calorimeter data were

considered to be corroborative inasmuch as the average value of the simu-

lated skin measurements was in excellent agreement with the average of

the ten highest calorimetric determinations, and within one standard de-

viation of the average of all thirty apparently reliable determinations.

Temperature rise at depth was measured routinely at the exposure

time of 3 seconds. Measurements were made under slabs of silicone

rubber of three different thicknesses. At the same time computations of

the temperature rise at depth, at the interface, and at the surface were made

for the appropriate thicknesses of the two layers. It was found that the

equations given yielded a slightly higher temperature rise at depth than

at the interface, an obvious physical impossibility; therefore, a proof of

the derivation was performed.[*]

The analysis showed that the equations were correct but that the

term, γ, was incorrect. Instead of:

$$\gamma = \frac{k_2 S_2 + k_1 S_1}{k_2 S_2 - k_1 S_1}$$

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[*] For this analysis and the consequent corrections, the authors are most

grateful to Eugene Toll of the Aeronautical Computer Laboratory, U. S.

Naval Air Development Center, Johnsville, Pa.

this value should have been:

$$\gamma = \frac{k_2 \, S_2 + \sqrt{k_1 \, S_1 \, k_2 \, S_2}}{k_2 \, S_2 - \sqrt{k_1 \, S_1 \, k_2 \, S_2}}$$

With the incorporation of this correction the theoretical temperature rise

at the interface became appreciably higher than that at depth as expected.

It was then possible to compare the experimental results with the theo-

retical values.

Table II presents this comparison for the temperature rise measured

within the backing layer and the values obtained by the corrected Equa-

tion (1) using the measured flux of 1.73 cal/cm^2 sec.

TABLE II

Thickness of Rubber Layer (mm)	No. of Measurements	Temperature Rise (°C)	
		Measured ± σ	Theoretical (°C/H x 1.73)
0.95	17	10.98 ± 0.40	10.90
0.55	12	28.13 ± 0.84	28.20
0.52	13	30.09 ± 0.55	29.93

It is seen that these values are in excellent agreement, the averages of

the experimental results being almost identical with the theoretical values.

The complete analysis of the temperature relationships is illustrated

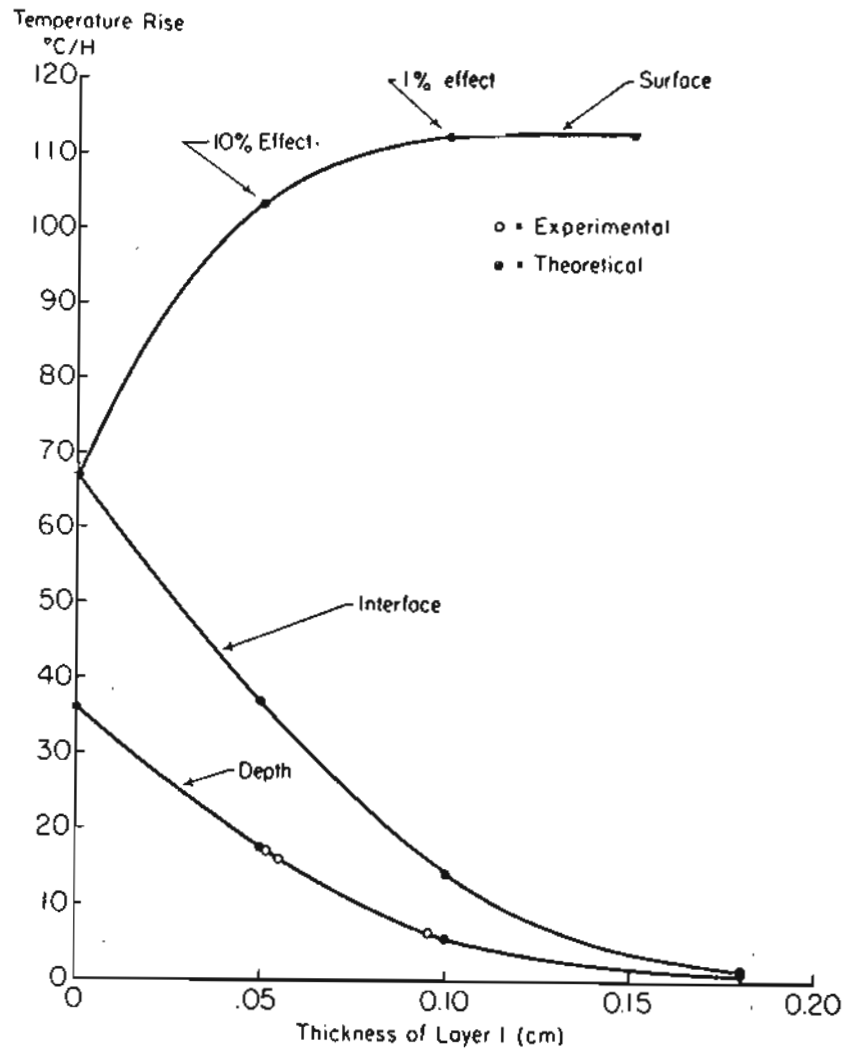in Figure 1 in which the temperature rises at depth within the simulated

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Figure 1. Temperature rise per unit heat flux at surface, interface and depth within layer 2 (simulated skin) and comparison of experimental and theoretical data.

skin, at the interface, and at the surface of the silicone rubber are

plotted against the thickness of the rubber layer (α) for the exposure

time of 3 seconds. The comparison of experimental and theoretical data

is shown graphically in the bottom curve, that of temperature rise at

depth. From the surface temperature rise curve it is seen that the

"critical thickness" occurs at about 0.1 cm. At greater thicknesses the

temperature rise as obtained from Eq. 2 coincides precisely with the

values obtained from Eq. 4, the well-known expression for the tempera-

ture rise of the surface of a homogeneous wall. At lesser thicknesses the

effect of the underlying layer is obvious in the depression of the surface

temperature rise. Extrapolation of this curve to zero thickness yields

the surface temperature rise of the simulated skin material as given

(3, 5) and as may be calculated from Eq. 4 on substitution of the thermal

properties of this layer. Furthermore, during the check on the deriva-

tion of the equations, it was found that the surface temperature rise of

the bare surface of the second layer could be obtained from the surface

temperature rise of the first layer at critical thickness or greater if

the thermal properties of both layers are known. This end could be

accomplished by inclusion in Eq. 4 of the next term of the expansion of

Eq. 3, viz., a multiplier $\left(\dfrac{\gamma - 1}{\gamma + 1} \right)$, expressive of the ratio of the

thermal properties of the two layers. Thus, for these materials, where

$$0.598 = \frac{\gamma - 1}{\gamma + 1}$$

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and $112.4^{\circ}C/_H$ = temperature rise of silicone rubber surface at critical

thickness or greater fro both Eq. 2 and Eq. 4, then 112.4 x 0.598 =

$67.2^{\circ}C/H$, the temperature rise of the bare simulated skin surface as

given and as calculated from Eq. 4 on substitution of the properties of the

simulated skin itself. Finally, the curve for the interface temperature

rise, an extrapolation to zero thickness, passes through this point also,

completing the confirmation of total internal agreement among the mathe-

matical expressions.

SUMMARY AND CONCLUSION

Experimental validation studies of the equations of Griffith and

Horton for transient heat flow through a two layer assembly (2) were

conducted with thin silicone rubber slabs overlaid on a simulated skin of

known thermal properties. This study resulted in important revisions in

the equations by correction of an error in the derivation of one of the

constants embodying the thermal properties of materials of both layers.

With the incorporation of these corrections, complete agreement was

attained between theoretical and experimental values and between values

computed by two different forms of the basic derivation.

It is concluded, therefore, that these corrected equations comprise

a rigorous representation of the thermal events and may be used with

confidence in solving problems which satisfy the boundary conditions

specified. In particular, these equations are applicable to the immediate

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problem for which they were sought, viz., studies of temperature rise under experimental fabrics on flame contact, and of the thermal properties of these fabrics. It is expected, furthermore, that they will be of inestimable value in solving physiological problems of heat flow into living skin where temperature at a shallow depth cannot be measured accurately.

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4. Investigation of the thermodynamic properties of the Cowbex-ADL Natural Gas Burner. Arthur D. Little, Inc., C-63206, June 15, 1961. (Final report prepared for the Amer. Gas Association.)

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Equation I- TEMPERATURE RISE AT DEPTH IN LAYER 2

$$U_2 = \frac{2H\lambda\sqrt{D_1}}{\gamma} \sum_{n=0}^{n=\infty} \left(-\frac{1}{\gamma}\right)^n \left\{ 2\sqrt{\frac{D_2 t}{\pi}} \, e^{-\left\{x-a\left[1-\sqrt{D_2/D_1}\,(2n+1)\right]\right\}^2/4D_2 t} \right.$$

$$\left. -\left[x-a\left\{1-\sqrt{D_2/D_1}(2n+1)\right\}\right]\left(1-\text{erf}\,\frac{x-a\left\{1-\sqrt{D_2/D_1}(2n+1)\right\}}{2\sqrt{D_2 t}}\right)\right\}$$

where

subscript 1 refers to top layer 1, subscript 2 refers to base layer 2

and U = Temperature rise

H = Heat flux perpendicular to surface

X = Total thickness from surface to point of temperature rise measurement

a = Thickness of layer 1

D = Thermal diffusivity = k/S

k = Thermal conductivity

S = Volume specific heat (density x specific heat)

$$\gamma^* = \frac{k_2 S_2 + \sqrt{k_1 S_1 k_2 S_2}}{k_2 S_2 - \sqrt{k_1 S_1 k_2 S_2}}$$

$$\lambda = (k_2\sqrt{D_1} - k_1\sqrt{D_2})^{-1}$$

t = time

*As corrected

Equation 2 - TEMPERATURE RISE AT SURFACE

$$U_0 = \frac{H}{k_1}\left[2\sqrt{\frac{D_1 t}{\pi}} - \frac{4}{\gamma}\sum_{n=0}^{n=\infty}\left(-\frac{1}{\gamma}\right)^n\left\{\sqrt{\frac{D_1 t}{\pi}}\ e^{-\alpha^2\frac{(n+1)^2}{t D_1}}\right.\right.$$

$$\left.\left. -\alpha(n+1)\left(1-\operatorname{erf}\frac{\alpha(n+1)}{\sqrt{D_1 t}}\right)\right\}\right]$$

Equation 3 - TEMPERATURE RISE AT INTERFACE

$$U_s = \frac{H}{k_1}\sum_{n=0}^{n=\infty}\left(-\frac{1}{\gamma}\right)^n\left(1-\frac{1}{\gamma}\right)\left[2\sqrt{\frac{D_1 t}{\pi}}\ e^{-\left(\frac{\alpha(2n+1)}{2\sqrt{D_1 t}}\right)^2}\right.$$

$$\left. -\alpha(2n+1)\left(1-\operatorname{erf}\frac{\alpha(2n+1)}{2\sqrt{D_1 t}}\right)\right]$$

Equation 4 - TEMPERATURE RISE AT SURFACE OF
SEMI-INFINITE HOMOGENEOUS SOLID

$$U_0 = 2H\sqrt{(t/S_1 k_1 \pi)} = (2H/k_1)\sqrt{(D_1 t/\pi)}$$